Optimal Tariffs in the Melitz Model: A Sufficient Statistics Approach for Trade Policy*

Tomohiro Ara†

Fukushima University

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Abstract

This paper shows that the variable nature of the trade elasticity provides new policy implications for optimal tariffs. To achieve the goal, we develop a heterogeneous firm model with (*i*) a general productivity distribution so that the trade elasticity is bilateral-specific to country-pairs; (*ii*) no outside good so that the wage rate is endogenous; and (*iii*) import tariffs so that tariff revenues are one of the welfare components. In this general setting, we find that optimal levels of import tariffs are the same across different trade models with a constant trade elasticity, conditional on the two sufficient statistics for welfare—the domestic trade share and the trade elasticity. However, the equivalence of optimal tariffs across different trade models no longer holds when the trade elasticity differs across markets. Calibrating the model to US data, optimal tariffs with a variable trade elasticity are substantially lower than those with a constant trade elasticity. Moreover, using analytical solutions of comparative statics, the effect of market size on optimal tariffs is quantitatively much smaller than that of variable trade costs.

Keywords: Optimal tariffs, variable trade elasticity, trade liberalization, market size.
JEL Classification Numbers: F12, F13, F16

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†Faculty of Economics and Business Administration, Fukushima University, Fukushima 960-1296, Japan. *Email address*: tomohiro.ara@gmail.com

1 Introduction

Gains from trade can be calculated by the only two sufficient statistics—the response of trade flows to changes in trade costs and the share of domestic expenditure—in a large class of trade models (Arkolakis et al., 2012). Stimulated by the theoretical results, recent empirical research has estimated the two sufficient statistics and substantiated significant heterogeneity in these measures that systematically vary with country characteristics. The first element, the "trade elasticity," tends to differ depending on whether country-pairs are proximate or distant, and large or small. For example, Bas et al. (2017) find that the trade elasticity is smaller for proximate country-pairs where the trade volume is already large. The second element, the "domestic trade share," also tends to be considerably affected by trade environments, in the sense that the domestic trade share is higher, the larger and the *less* open are countries.[1] These pieces of empirical evidence imply that trade liberalization and market size may have an different effect on the two sufficient statistics for welfare gains from trade. Thus we need to take into account country-pair differences in these statistics to correctly expect the impact of trade, such as reduction in trade costs and expansion in market size through trade agreements.

How does the fact that the two sufficient statistics vary with country-pairs matter for optimal trade policy? To address this key question, we develop an asymmetric-country version of the Melitz (2003) model with CES preferences and monopolistic competition. One of the drawbacks in this framework is that when productivity is Pareto distributed—one of the most commonly used productivity distributions in the literature, the trade elasticity is *unique* to any country-pairs, irrespective of country characteristics. Moreover, firms' markups are constant which implies, under firm heterogeneity, that market size has no effect on the domestic trade share via selection. To circumvent these limitations and provide more realistic policy implications, we make three main departures from existing work. First, we work with a general productivity distribution that generates a trade elasticity *bilateral-specific* to country-pairs. Second, we consider endogenous wages that restore the role of market size in the domestic trade share via selection. Finally, we analyze not only iceberg trade costs but also import tariffs that a government chooses so as to maximize welfare. These distinctions jointly help us to understand the different effects of competitive pressures on the sufficient statistics and address its consequence for optimal trade policy in a single unified setting.

Our starting point is to note that not only do trade costs but also market size has a critical effect on wages. Trade costs have been steadily declining over time by both technological improvements and trade negotiations. For example, Hummels (2007) finds that the measure of international air transport prices per ton has fallen more than ten times worldwide between 1955 and 2004 due largely to the adoption of jet engines; similarly continuous effort by the World Trade Organization (WTO) has decreased worldwide average tariffs from 8.6 percent to 3.2 percent between 1960 and 1995, greatly increasing wages of trading countries. On the other hand, the significance of changes in market size is best demonstrated with an example. Figure 1 displays the transition in population and GDP per capita as a measure of market size and wages, respectively. Panel A shows the case of the United States, indicating a clear monotone relationship between population and GDP per capita. In contrast, Panel B shows the case of Japan where population is gradually declining due mainly to the low birthrate. According to the Cabinet Office of Japan, the population is expected to decrease from 124 million in 2020 to 97 million in 2050 and to 86 million in 2060. It is often said that gradient shrinking in its domestic market size together with heavy reliance on overseas demand could force Japan to see a steep decline in GDP per capita (Nikkei Asia, 2019). This shows that changes in both trade costs and market size were significant over the last decades, critically affecting wages and hence national welfare.

[1] For the trend in the domestic trade share, see Eaton and Kortum (2002, 2011) with aggregate data, and Bernard et al. (2007) and Mayer and Ottaviano (2008) with firm-level data.

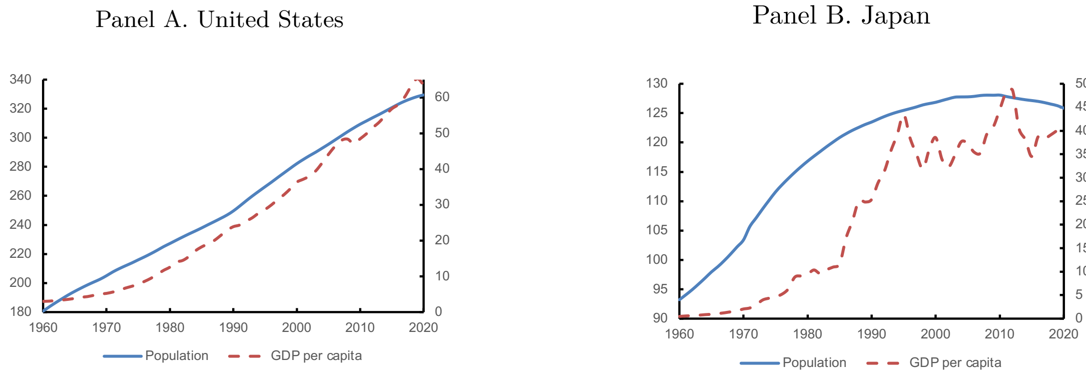

Figure 1: Population and GDP per capita during 1960–2020

Source: World Bank Data and the author's construction.
Note: The left (right) scale measures population in units of millions (GDP per capita in units of thousand US dollar).

We show that if wages are endogenous, a large country entails *weak* domestic selection by which to allow inefficient firms to survive in a domestic market. To see this reason, consider unilateral reduction in trade costs which has two effects on firms' expected profits. First, such liberalization directly decreases expected profits by reducing markups on foreign goods. Second, it indirectly increases expected profits by reducing wages, i.e., production costs of firms, which occurs to recover the trade balance (Demidova and Rodríguez-Clare, 2013). In contrast, unilateral expansion in market size has no direct effect on expected profits via markups under CES preferences and monopolistic competition, while it indirectly decreases expected profits by raising wages due to the home market effect (Krugman, 1980). As increased wages cause higher production costs and lower profitability, firm entry is relocated from an expanding country with higher wages to a non-expanding country with lower wages. This relocation allows inefficient firms to survive in a domestic market.

Weak domestic selection associated with increased wages may account for the shift in trade patterns that Japan experienced in the late 1990s. For example, using firm-level data on manufacturing sectors in Japan, Fukao et al. (2008) study how firms' productivity differences affected firms' turnovers between 1990 and 2003. They find that the turnover rate was significantly higher for less productive firms, but nearly a half of the top 10 percent of the most productive firms also exited. This puzzling fact can be explained by increased wages in Japan, which induced these most productive firms to seek for cheaper labor in foreign markets such as China, while simultaneously allowing less productive firms to survive in the domestic market in Japan.

Our selection effect of market size contradicts the finding in Melitz and Ottaviano (2008). The key reason comes from the presence of an outside good that makes wages exogenously fixed in their model. In that case, the difference in market size leads to trade patterns such that a large (resp. small) country specializes in the differentiated (resp. outside) good sector. As a result, the larger is market size, the tougher is competition in the differentiated good sector, forcing the least efficient firms to exit. If this outside good is absent, however, wages are endogenously determined by the trade balance and the difference in market size does not allow for the trade patterns via changes in wages. Thus it is not surprising that the effect of market size on selection is different from Melitz and Ottaviano (2008).[2] Although a large country suffers from weak domestic selection, it can nonetheless enjoy welfare gains from its market size since a negative impact on domestic selection may be dominated by a positive impact on product variety.

[2]Our paper also differs from Melitz and Ottaviano (2008) in consumer preferences that generate constant or variable markups; however, the absence of an outside good can reverse their results even with variable markups (Demidova, 2017).

Given that endogenous wages give rise to the different effects of trade costs and market size on selection, what can we say about its policy implications? In analyzing optimal trade policy, we show that the difference is crucial for the characterization of optimal tariffs, i.e., the welfare-maximizing tariffs that each country would impose without fearing retaliation. In the present model, the optimal tariffs are inversely related to a trading partner's export supply elasticity, which is composed of both the domestic trade share and the trade elasticity, as in existing models. We find, however, that reduction in trade costs and expansion in market size do not necessarily lead to high optimal tariffs in our model. From a policy point of view, the effect of market size on optimal tariffs is of particular interest: a large country does not always benefit from high tariffs. Our model shows that a large country can enjoy a terms-of-trade gain from setting tariffs as in the conventional optimal tariff theory, but it also suffers from a welfare loss from weak domestic selection where tariffs accelerate this loss by protecting inefficient firms from foreign competition. From this tradeoff associated with selection, whether the former benefit of tariffs dominates the latter cost depends on whether the trade elasticity is constant or variable. If the trade elasticity is variable and differs across markets as reported by recent empirical work,[3] optimal tariffs can decline with market size through an endogenous response in the trade elasticity.

To help better appreciate the policy result, following Chaney (2008), let us decompose the trade elasticity into the intensive margin elasticity and the extensive margin elasticity where the former refers to the elasticity of each incumbent firm's shipment whereas the latter refers to the elasticity of new entrants' shipment. Since the intensive margin elasticity is constant under CES preferences and monopolistic competition, the variable nature of the trade elasticity should come from the extensive margin elasticity, which in turn depends on the micro structure of the model. In the homogeneous firm model where all firms export, there is no adjustment margin from new firms' entry (i.e., the extensive margin elasticity is zero) and hence the trade elasticity is the same as the intensive margin elasticity. In the heterogeneous firm model where productivity is drawn from a Pareto distribution, the extensive margin elasticity is constant and so is the trade elasticity (Chaney, 2008). In these special cases, market size has no effect on the trade elasticity and the optimal tariffs always increase with market size only through the domestic trade share (Gros, 1987; Felbermayr et al., 2013). In general cases where the productivity distribution is unrestricted, however, the theoretical result that the trade elasticity is constant does not hold. Importantly, empirical work has found substantial variation in the trade elasticity across country-pairs, which cannot be rationalized in most of previous models. Due to this additional channel, we find that optimal tariffs do not necessarily increase with market size.

While this paper mainly analyzes the qualitative aspect of optimal tariffs with a variable trade elasticity, our analytical results also allow us to quantitatively measure a discrepancy in optimal tariffs that can arise when the trade elasticity is assumed constant despite that the "true" trade elasticity is variable. Our model calibrated to US data indeed reveals that optimal tariffs with a variable trade elasticity are substantially lower than those with a constant trade elasticity. In our numerical exercise, levels of optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model. The difference is accounted for by the fact that the trade elasticity has been implicitly assumed to be constant in the literature. Using analytical solutions of comparative statics, we also find that the effect of market size on optimal tariffs is quantitatively much smaller than that of variable trade costs. This quantitative result is consistent with our theoretical prediction that a large country may not always benefit from setting high tariffs. We also discuss how our calibration of optimal tariffs using US data has the practical relevance to real-world trade policies in the United States, such as recent tariff hikes under the Trump administration, offering useful insights for policymakers.

[3] See, for example, Helpman et al. (2008), Novy (2013), Spearot (2013) and Bas et al. (2017). Our theoretical approach is close to Helpman et al. (2008) and Bas et al. (2017) who rest on CES preferences and monopolistic competition to provide evidence.

A number of papers have explored welfare and policy implications in the homogeneous and heterogeneous firm models. Regarding welfare implications, Arkolakis et al. (2012) derive a simple formula that can capture welfare gains only by the domestic trade share and the trade elasticity. As this applies to a surprisingly large set of trade models, followup papers have examined extension/robustness of the welfare results. For example, Arkolakis et al. (2019) study general demand functions that yield variable markups, Felbermayr et al. (2015) introduce tariffs that raise government revenues, and Head et al. (2014) and Melitz and Redding (2015) consider a non-Pareto distribution that makes the trade elasticity variable. We show that the Arkolakis et al. (2012) welfare formula is also applicable to the conventional wisdom of optimal tariffs. In particular, conditional on their sufficient statistics for welfare, the optimal level of import tariffs is the same across different trade models with a constant trade elasticity, but, more generally, the level depends on the micro structure that makes the trade elasticity variable.[4] We also find that firm heterogeneity drawn outside a Pareto distribution can affect a welfare measurement as in Head et al. (2014) and Melitz and Redding (2015); however, the scope of this paper differs from theirs since we analytically show a new optimal tariff formula with a variable trade elasticity and quantitatively investigate the effect of the two major sources of competitive pressures on optimal trade policy. Moreover, like original work by Melitz (2003), they mainly consider trade between symmetric countries and hence cannot distinguish bilateral and unilateral effects of exogenous shocks on optimal trade policy.

As for policy implications, there is a large literature of optimal tariffs. Gros (1987) derives optimal tariffs in the homogeneous firm model which is inversely related to the trade elasticity and the domestic trade share of a trading partner. Using Ossa (2011)'s framework featured with tariff-induced production relocation effects, Ossa (2014) provides a comprehensive analysis of optimal tariffs in a multi-sector, general-equilibrium model which nests the traditional (terms-of-trade), new trade (profit-shifting) and political-economy motives in the homogeneous firm model. These analyses of optimal tariffs are extended to the heterogeneous firm model by Demidova and Rodríguez-Clare (2009) for a small economy and Felbermayr et al. (2013) for a large economy. In so doing, Felbermayr et al. (2013) show that optimal tariffs are lower in the heterogeneous firm model than the homogeneous firm model, holding the domestic trade share equal. While existing work contributes to our understanding of optimal trade policy, one of the limitations is that the trade elasticity is constant in either the homogeneous or heterogeneous firm model. However, the existence of a constant trade elasticity is highly sensitive to parameter restrictions, and welfare changes can be mis-estimated when the "true" trade elasticity is variable (Melitz and Redding, 2015). In the context of trade policy, the optimal level of import tariffs can be mis-estimated when the same parameter restrictions are imposed. We highlight this key caveat not only by analytically characterizing optimal tariffs with a variable trade elasticity, but also by quantitatively measuring these magnitudes from our model and existing models calibrated to US data.

Recently, Costinot et al. (2020) offer a strict generalization of Gros (1987) in the homogeneous firm model, and Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2013) in the heterogeneous firm model with a Pareto distribution. They find that when tariffs are uniform across all firms, optimal tariffs can be lower in the heterogeneous firm model than in the homogeneous firm model due to non-convexity of aggregate goods across domestic and foreign markets. In contrast, we show that the same result can arise due to variability of the trade elasticity across domestic and foreign markets. Although their new element in optimal tariffs is closely related to ours in the sense that both arise in the presence of selection, our element is relatively easy to measure from the firm-level data which is in turn directly applicable to the quantification of optimal tariffs. In that respect, we investigate a different but complementary channel in optimal trade policy.

[4] Our sufficient statistics approach to optimal trade policy is related to that by Lashkaripour (2021) in that such policy can be calculated by the welfare formula by Arkolakis et al. (2012). One of the critical differences is that his baseline analysis builds on the Ricardian setup of Eaton and Kortum (2002), which means that the trade elasticity is constant at the supply side parameter.

2 Setup

In this section, we first describe the behaviors of consumers, firms and governments that play a crucial role in the model. Then we define the equilibrium conditions that pin down key endogenous variables of the model.

2.1 Basics

There are two countries indexed by i, j that use only labor to produce differentiated goods in a single sector. Country i is populated by a mass L_i of identical consumers whose preferences are given by

$$U_i = \left(\sum_{n=i,j} \int_{\omega \in \Omega_n} q_{ni}(\omega)^{\rho} d\omega \right)^{1/\rho}, \qquad 0 < \rho < 1,$$

where an elasticity of substitution between varieties is $\sigma = 1/(1-\rho) > 1$. Throughout this paper, we denote the exporting (importing) country by the first (second) subscript; thus $q_{ji}(\omega)$ is a quantity shipped from country j to country i of variety ω. Let $p_{ji}(\omega)$ denote a price of the corresponding variety. As consumers derive utility by consuming both domestic and foreign varieties, the budget constraint is expressed as $\sum_n \int_\omega p_{ni}(\omega) q_{ni}(\omega) d\omega$. Consumer utility maximization subject to this constraint yields the following demand:

$$q_{ji}(\omega) = R_i P_i^{\sigma-1} p_{ji}(\omega)^{-\sigma},$$

where $P_i = \left(\sum_n \int_\omega p_{ni}(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}$ is a price index associated with an aggregate good defined as $Q_i \equiv U_i$ and $R_i = P_i Q_i$ is aggregate expenditure.

Firm behavior is similar to that modeled by Melitz (2003). Upon paying fixed entry costs f_i^e, a mass M_i^e of entrants randomly draw productivity φ from a distribution $G_i(\varphi)$. The distribution has support $(\varphi_{\min}, \varphi_{\max})$ where the upper bound is either finite ($\varphi_{\max} < \infty$) or infinite ($\varphi_{\max} = \infty$). After observing productivity φ, each firm decides whether to exit or produce. If a firm from country i chooses to produce for country i's market, it incurs fixed overhead costs f_{ii} and constant marginal costs that are inversely proportional to productivity: $l_{ii}(\varphi) = f_{ii} + q_{ii}(\varphi)/\varphi$. On the other hand, if a firm from country j chooses to produce for country i's market, it incurs variable trade costs θ_{ji} and fixed trade costs f_{ji} of the same cost function: $l_{ji}(\varphi) = f_{ji} + \theta_{ji} q_{ji}(\varphi)/\varphi$. We assume $\theta_{ji} > 1$ (with $\theta_{jj} = 1$) and all the production costs are measured in a source country's labor units. For example, a firm from country j to country i incurs $w_j l_{ji}(\varphi)$ where w_j is a wage rate in country j.

A government imposes ad-valorem tariffs $\tau_{ji} = 1 + t_{ji} > 1$ (with $\tau_{jj} = 1$ and $t_{jj} = 0$) on foreign goods. While tariffs change the price of these goods, they can be imposed either *before* or *after* firms charge markups, which are known as *cost* or *demand* shifters of tariffs. In this paper, tariffs are modeled as demand shifters that directly affect consumer demand of foreign goods.[5] Hence, a firm from country j to country i receives only the net-of-tariff price $p_{ji}(\varphi)/\tau_{ji}$ (that is, $p_{ji}(\varphi)$ is the tariff-inclusive price charged in country i) whereby a government in country i collects tariff revenues $(\tau_{ji} - 1) p_{ji}(\varphi)/\tau_{ji}$ from that firm.

This completes the description of consumer preferences, firm technology and government policy analyzed in the paper. From the model setting, the firm earns the following profits:

$$\pi_{ji}(\varphi) = \left(\frac{p_{ji}(\varphi)}{\tau_{ji}} - \frac{\theta_{ji} w_j}{\varphi} \right) q_{ji}(\varphi) - w_j f_{ji}.$$

[5] On the other hand, tariffs as cost shifters directly affect marginal costs of foreign firms. Felbermayr et al. (2015) show that the distinction of two forms of tariffs—though evidence is rare—can be quantitatively important for welfare.

2.2 Equilibrium in Levels

As usual in monopolistic competition with a large number of firms, the pricing decision of any one firm has a negligible effect on other firms' behaviors. Each firm thus sets the price to maximize the profits taking as given the price index and aggregate expenditure in consumer demand. Since the firm from country j to country i with productivity φ incurs marginal costs $\theta_{ji} w_j/\varphi$ as well as tariffs τ_{ji}, firm profit maximization means that the optimal pricing is to charge a constant markup $\sigma/(\sigma-1) = 1/\rho$ over these costs:

$$p_{ji}(\varphi) = \frac{\tau_{ji}\theta_{ji}w_j}{\rho\varphi}.$$

Using this optimal pricing for $\pi_{ji}(\varphi)$ and defining the firm revenue net of tariffs as $r_{ji}(\varphi) \equiv p_{ji}(\varphi)q_{ji}(\varphi)/\tau_{ji}$, the firm variable profits are expressed as $r_{ji}(\varphi)/\sigma$, which are strictly increasing in productivity φ. Given that, there is a unique productivity cutoff at which an exporting firm makes zero profits, namely, $r_{ji}(\varphi^*_{ji})/\sigma = w_j f_{ji}$. This is referred to as the **zero cutoff profit (ZCP) condition** (see Appendix A.1):

$$B_i \tau_{ji}^{-\sigma}(\theta_{ji}w_j)^{1-\sigma}(\varphi^*_{ji})^{\sigma-1} = \sigma w_j f_{ji}, \tag{1}$$

where $B_i \equiv R_i(\rho P_i)^{\sigma-1}$ is the index of market demand. Note that (1) also pins down the domestic productivity cutoff φ^*_{jj} when $i = j$. We restrict attention to the case where selection into exporting occurs, i.e., $\varphi^*_{ji} > \varphi^*_{jj}$. Using (1), we can easily show that the selection condition holds when trade costs are sufficiently large and market size is not too different, where the latter implies that relative market demand B_i/B_j—proportional to relative market size measured by R_i/R_j—is not too large or too small.

Free entry requires that the expected profits earned from all operating countries equal the fixed entry costs. Following Melitz (2003), let $J_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] dG_i(\varphi)$. From the definition of φ^*_{ij} in view of (1), any firms from country i to country j earn the expected profits $w_i f_{ij} J_i(\varphi^*_{ij})$ (see Appendix A.1). As described above, however, any entrants in country i must incur the fixed entry costs $w_i f^e_i$ before drawing productivity. Taken together, the **free entry (FE) condition** is expressed as

$$\sum_{n=i,j} f_{in} J_i(\varphi^*_{in}) = f^e_i. \tag{2}$$

The FE condition determines the market demands B_i, B_j by adjusting the price indices P_i, P_j in equilibrium, so that potential entrants make zero expected profits.

Labor is used for both entry and production, which must equal aggregate labor supply in the economy. From (1) and (2), the **labor market clearing (LMC) condition** is expressed as (see Appendix A.2)

$$\frac{R_i - T_i}{w_i} = L_i,$$

where $T_i \equiv (\tau_{ji} - 1)R_{ji}$ is aggregate tariff revenue and R_{ji} is aggregate expenditure on country j's goods in country i.[6] The LMC condition pins down the wage rates w_i, w_j, which can be better understood by rewriting as $R_i = w_i L_i + T_i$: country i's wage w_i is determined by the equality between aggregate expenditure R_i and aggregate labor income $w_i L_i$ plus aggregate tariff revenue T_i. There is no net surplus other than $w_i L_i$ and T_i because free entry drives down expected net profits to zero in equilibrium.

[6] As $r_{ji}(\varphi)$ is defined as net of tariffs, R_{ji} is also defined as net of tariffs, aggregating $r_{ji}(\varphi)$ over productivity $\varphi \in (\varphi^*_{ji}, \varphi_{\max})$ among a mass M^e_i of entrants (see Appendix A.2 for a precise definition). Thus, R_i is aggregate expenditure net of tariffs.

To work on general equilibrium, we relate the LMC condition with the **trade balance (TB) condition**. While the TB condition requires $R_{ij} = R_{ji}$, it is equivalent to the LMC condition, in that the two conditions yield the same equality, $R_i = w_i L_i + T_i$ (see Appendix A.2). Notice that aggregate expenditure in country i consists of expenditures on domestic goods in country i and foreign goods from country j inclusive of tariffs ($R_i = \sum_n \tau_{ni} R_{ni}$), because consumers access foreign goods inclusive of tariffs. On the other hand, aggregate labor income in country i consists of revenues by domestic firms and exporting firms in country i net of tariffs ($w_i L_i = \sum_n R_{in}$), because firms earn foreign revenues net of tariffs. Let $\lambda_{ji} \equiv \tau_{ji} R_{ji} / \sum_n \tau_{ni} R_{ni}$ denote the foreign trade share in country i inclusive of tariffs. Similarly, let $\tilde{\lambda}_{ji} \equiv R_{ji} / \sum_n R_{ni}$ denote the corresponding trade share net of tariffs. Then, we can express aggregate expenditures on domestic goods and foreign goods by the expenditure shares net of tariffs: $R_{ii} = \tilde{\lambda}_{ii} w_i L_i,\ R_{ji} = \tilde{\lambda}_{ji} w_i L_i$.[7] Plugging these into the LMC condition, we get a familiar expression of the TB condition that applies to the presence of tariffs:

$$w_i L_i = \sum_{n=i,j} \tilde{\lambda}_{in} w_n L_n. \tag{3}$$

Now we are ready for characterizing the equilibrium variables when countries have an option to set tariffs. For given levels of exogenous variables, **equilibrium in levels** is defined as a set of the vector $\{\varphi_{ii}^*, \varphi_{ij}^*, B_i, w_i\}$ which are jointly characterized by the system of eight equations in (1), (2), and (3) for i, j. By Walras's law, wages in country j can be normalized to unity, i.e., $w_j = 1$. Once levels in these key variables are determined, other endogenous variables are written as a function of them. In particular, using the definition of B_i in (1), welfare per worker equivalent to the real wages is expressed as follows (see Appendix A.3):

$$W_i = \left(\frac{L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} (\mu_i)^{\frac{1}{\rho}} \rho \varphi_{ii}^*, \tag{4}$$

where $\mu_i \equiv R_i / w_i L_i$ is referred to as a "tariff multiplier" (Felbermayr et al., 2015) in the following analysis. The variable enters the welfare expression in (4), as tariff revenues are rebated back to consumers. This means that the tariff multiplier μ_i as well as the domestic trade share $\tilde{\lambda}_{ji}$ have a close relationship with tariffs τ_{ji}. From the definitions of these variables and $\mu_i = 1 + (\tau_{ji} - 1)\tilde{\lambda}_{ji}$ in view of $R_i = w_i L_i + T_i$, we get

$$\tilde{\lambda}_{ji} = \frac{\lambda_{ji}}{\tau_{ji}(1 - \lambda_{ji}) + \lambda_{ji}},$$
$$\mu_i = \frac{\tau_{ji}}{\tau_{ji}(1 - \lambda_{ji}) + \lambda_{ji}}.$$

Clearly, $\tilde{\lambda}_{ji} = \lambda_{ji}$ and $\mu_i = 1$ when country i does not impose tariffs on foreign goods from country j ($\tau_{ji} = 1$).

3 Impact of Competition

The last section has defined the equilibrium variables in *levels*. This section defines the equilibrium variables in *changes* for examining the effects of trade liberalization and market size. Though we will mainly focus on variable trade costs in this section, the effects of fixed trade costs and tariffs are very similar. In contrast to variable and fixed trade costs that use up real resources, however, tariffs raise government revenues and are used to manipulate the terms of trade. In Section 4, we will characterize welfare-maximizing optimal tariffs by taking account of this motive.

[7] The result follows from noting $w_i L_i = \sum_n R_{in} = \sum_n R_{ni}$ (from $R_{ij} = R_{ji}$) in the denominator of $\tilde{\lambda}_{ji}$.

3.1 Equilibrium in Changes

Suppose that country i unilaterally reduces variable trade costs θ_{ji} and expands market size L_i, holding all other exogenous variables constant. We are interested in the impact of competition induced by these changes, which may potentially differ between them. Let a "hat" denote proportional changes in a variable ($\hat{x} \equiv dx/x$). Taking the log and totally differentiating the ZCP condition (1),

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma\hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}. \tag{5}$$

Though we allow for changes in both θ_{ji} and L_i, (5) includes changes in only θ_{ji} because (1) does not directly involve L_i. Then (5) shows that changes in the export productivity cutoff $\hat{\varphi}^*_{ji}$ are linked with changes in the other endogenous variables and variable trade costs. Like (1), changes in the domestic productivity cutoff $\hat{\varphi}^*_{jj}$ are also expressed by (5) when $i = j$ by noting $\hat{\theta}_{jj} = 0$.

Similarly, totally differentiating the FE condition (2) and rearranging,

$$\hat{\varphi}^*_{ij} = -\alpha_i\hat{\varphi}^*_{ii}, \tag{6}$$

where

$$\alpha_i \equiv \frac{f_{ii}J_i'(\varphi^*_{ii})\varphi^*_{ii}}{f_{ij}J_i'(\varphi^*_{ij})\varphi^*_{ij}}.$$

As (2) does not directly involve θ_{ji} and L_i, changes in these variables do not enter (6). Looking at α_i, we have $\alpha_i > 0$ since $J_i(\varphi^*)$ is strictly decreasing in φ^*. Then (6) shows that changes in the export productivity cutoff $\hat{\varphi}^*_{ij}$ occur in opposite directions to those in the domestic productivity cutoff $\hat{\varphi}^*_{ii}$. Further, exploiting the facts that $J_i(\varphi^*)$ relates to expected profits and the TB condition requires $R_{ij} = R_{ji}$, we can show that α_i equals R_{ii}/R_{ji}, the ratio of aggregate expenditure on domestic and foreign goods in country i (see Appendix A.4). This in turn allows us to express the foreign trade shares and the tariff multiplier in terms of α_i:

$$\lambda_{ji} = \frac{\tau_{ji}}{\alpha_i + \tau_{ji}}, \quad \tilde{\lambda}_{ji} = \frac{1}{\alpha_i + 1}, \quad \mu_i = \frac{\alpha_i + \tau_{ji}}{\alpha_i + 1}.$$

Finally, using $\tilde{\lambda}_{ji}$ introduced above, rewrite the TB condition (3) as $w_iL_i/(\alpha_i + 1) = w_jL_j/(\alpha_j + 1)$ where α_i is a function of $\varphi^*_{ii}, \varphi^*_{ij}$. Taking the log and totally differentiating this equality and using (6),

$$\hat{w}_i - \hat{w}_j = -\beta_i\hat{\varphi}^*_{ii} + \beta_j\hat{\varphi}^*_{jj} - \hat{L}_i, \tag{7}$$

where

$$\beta_i \equiv \frac{\alpha_i}{\alpha_i + 1}[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i],$$

and $\gamma_{in} \equiv -d\ln\int_{\varphi^*_{in}}^{\varphi_{\max}} \varphi^{\sigma-1}dG_i(\varphi)/d\ln\varphi^*_{in}$ is the extensive margin elasticity that arises in firm heterogeneity.[8] (7) shows that changes in wages $\hat{w}_i, \hat{w}_j$ are linked with changes in domestic productivity cutoffs $\hat{\varphi}^*_{ii}, \hat{\varphi}^*_{jj}$ where only changes in market size enter. Looking at $\beta_i (> 0)$, we find that the changes operate through two channels. First is the *intensive* margin: exogenous shocks induce incumbent firms to adjust their production through changes in consumer demand with an elasticity of $\sigma - 1$ under CES preferences. Second is the *extensive* margin: exogenous shocks induce entry of new firms or exit of incumbent firms through changes in competitiveness of each market with an elasticity of γ_{in} under firm heterogeneity.

[8] See Arkolakis et al. (2012, p.110) where firm productivity is measured by unit labor requirements that are the *inverse* of φ. We define the extensive margin elasticity with a minus sign so that $\gamma_{in} > 0$ (like the intensive margin elasticity $\sigma - 1 > 0$).

Now we are ready for characterizing the equilibrium variables in changes. Just as (1), (2) and (3) are used to solve for equilibrium in levels, (5), (6) and (7) are used to solve for equilibrium in changes. Specifically, **equilibrium in changes** is defined as a set of the vector $\{\hat{\varphi}_{ii}^*, \hat{\varphi}_{ij}^*, \hat{B}_i, \hat{w}_i\}$ which are jointly characterized by the system of eight equations in (5), (6), and (7) for i, j. By Walras's law, changes in country j's wage can be normalized to zero, i.e., $\hat{w}_j = 0$. Once changes in these key variables are determined, changes in other endogenous variables are written as a function of them. In particular, taking the log and totally differentiating (4), changes in welfare per worker are expressed as follows (see Appendix A.5):

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}_{ii}^* + \frac{\hat{L}_i}{\sigma - 1}. \tag{8}$$

(8) means that the domestic productivity cutoff φ_{ii}^* is a sufficient statistic for welfare even with tariff revenues. Any exogenous shocks give rise to changes in welfare through changes in the cutoff φ_{ii}^* that directly relate to resource reallocations à la Melitz (2003). In the presence of tariffs, such shocks also induce changes in welfare through changes in the tariff multiplier μ_i that relate to tariff revenue rebated back to consumers. However, the first term of (8) shows that welfare changes associated with these effects are summarized by changes in φ_{ii}^*. This impact applies not only to variable trade costs but also to market size, because the cutoff is endogenously affected through changes in wages (see (1)). On top of that, changes in market size have an additional channel for welfare through changes in product variety, given in the second term of (8).

3.2 Trade Costs

In Section 3.1, we have characterized equilibrium in changes by trade costs and market size at the same time. While this enables us to study these two effects using single equations, most models examine them separately. Demidova and Rodríguez-Clare (2013), for example, consider a welfare effect of asymmetric trade liberalization, keeping market size constant. They show that when wages are endogenously determined by the TB condition, unilateral trade liberalization increases welfare in a liberalizing country. Their result implies that the impact of trade liberalization is reversed, as such liberalization is known to decrease welfare in a liberalizing country when wages are exogenously fixed by an outside good (Demidova, 2008; Melitz and Ottaviano, 2008). Here, we provide analytical solutions of Demidova and Rodríguez-Clare (2013)'s results.[9]

We can extract the impact of unilateral trade liberalization on equilibrium by keeping market size constant. Recall that the model has the system of eight equations ((5), (6), (7)) for eight unknowns ($\hat{\varphi}_{ii}^*, \hat{\varphi}_{ij}^*, \hat{B}_i, \hat{w}_i$ for i, j), where labor in country j is the numéraire. Solving these equations by setting $\hat{L}_i = 0$,

$$\begin{aligned} \hat{\varphi}_{ii}^* &= -\frac{\rho(\beta_j + \rho)}{\Xi} \hat{\theta}_{ji}, \\ \hat{\varphi}_{jj}^* &= -\frac{\rho(\beta_i - \rho\alpha_i)}{\Xi} \hat{\theta}_{ji}, \\ \hat{w}_i &= \frac{\rho^2(\beta_i + \alpha_i\beta_j)}{\Xi} \hat{\theta}_{ji}, \end{aligned} \tag{9}$$

where $\beta_i - \rho\alpha_i > 0$ (from the definitions of α_i and β_i) and $\Xi \equiv \prod_n(\beta_n + \rho) - \prod_n(\beta_n - \rho\alpha_n) > 0$. (9) shows that reduction in θ_{ji} increases $\varphi_{ii}^*, \varphi_{jj}^*$ and decrease w_i. From (8), it then follows that welfare rises not only in country j but also in country i because a fall in w_i is smaller than a fall in P_i (i.e., the real wages w_i/P_i rise). Tariff revenues rebated back to consumers increase by raising μ_i, which additionally contributes to welfare.

[9] Though the results in this section are not entirely new, previous work has not provided the analytical solutions of the impact of trade costs under a general productivity distribution, which is shown to be useful in quantifying optimal tariffs in Section 5.

Intuition behind the results is clearly seen by solving (5) and (6) first without (7):

$$\begin{aligned} \hat{\varphi}_{ii}^{*} &= \frac{1}{\alpha_i\alpha_j - 1}\hat{\theta}_{ji} - \frac{\alpha_j + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i, \\ \hat{\varphi}_{jj}^{*} &= -\frac{\alpha_j}{\alpha_i\alpha_j - 1}\hat{\theta}_{ji} + \frac{\alpha_i + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i, \end{aligned} \tag{10}$$

where $\alpha_i\alpha_j - 1 > 0$. In (10), the first term is the direct effect of variable trade costs and the second term is the indirect effect through changes in wages.[10] The direct effect decreases expected profits in a liberalizing country by reducing markups on imports, but increase expected profits in a non-liberalizing country by allowing firms to export more easily. As a result, reduction in θ_{ji} deters entry in country i and induces entry in country j, decreasing φ_{ii}^{*} and increasing φ_{jj}^{*}. Note that this effect exists even when wages are fixed by an outside good. From (8), these changes imply that unilateral trade liberalization decreases welfare in a liberalizing country but increases welfare in a non-liberalizing country (Demidova, 2008; Melitz and Ottaviano, 2008).

If wages are endogenous, reduction in θ_{ji} leads to a rise in imports in country i which is counteracted by a fall in wages. The indirect effect increases expected profits in a liberalizing country by reducing production costs of firms, but decreases expected profits in a non-liberalizing country by raising production costs there relative to a liberalizing country. Hence, reduction in w_i (by reduction in θ_{ji}) induces entry in country i but deters entry in country j, increasing φ_{ii}^{*} and decreasing φ_{jj}^{*}. The indirect effect operates in opposite directions to the direct effect, but (9) shows that both φ_{ii}^{*} and φ_{jj}^{*} rise as a result of reduction in θ_{ji}. This equilibrium result holds only when the indirect effect outweighs the direct effect for φ_{ii}^{*} but the converse is true for φ_{jj}^{*}. Since the domestic productivity cutoff rises, (8) suggests that unilateral trade liberalization increases welfare in both countries (Demidova and Rodríguez-Clare, 2013). Moreover, since the export productivity cutoff falls (see (6)), such liberalization allows more firms to export and decreases the domestic trade share.

While we have focused on the impact of variable trade costs of importing θ_{ji}, the impact of *any* trade costs $(\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji})$ on productivity cutoffs is qualitatively similar (see Appendix A.6). In case of variable trade costs of exporting θ_{ij}, for example, we get

$$\begin{aligned} \hat{\varphi}_{ii}^{*} &= -\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ij}, \\ \hat{\varphi}_{jj}^{*} &= -\frac{\rho(\beta_i + \rho)}{\Xi}\hat{\theta}_{ij}, \\ \hat{w}_i &= -\frac{\rho^2(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\theta}_{ij}. \end{aligned}$$

Hence, reduction in export costs θ_{ij} also increases the domestic productivity cutoff in both countries as above. The only difference is that reduction in *import* costs θ_{ji} *decreases* w_i, whereas reduction in *export* costs θ_{ij} *increases* w_i. The difference in wage changes reflects the fact that reduction in θ_{ij} leads to a rise in exports in country i, which must be counteracted by a rise in wages in country i to recover the trade balance.

Finally, starting from a symmetric situation, welfare gains from unilateral trade liberalization are always greater in a liberalizing country than in a non-liberalizing country. Consider the effect of variable trade costs of importing θ_{ji}. Evaluating (9) at $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$ reveals that $|\hat{\varphi}_{ii}^{*}| > |\hat{\varphi}_{jj}^{*}|$, which implies that $\hat{W}_i > \hat{W}_j$ from (8) by setting $\hat{L}_i = 0$. Thus, reduction in θ_{ji} leads to greater welfare gains in country i than in country j. This also holds for variable trade costs of exporting θ_{ij}, in the sense that, starting from a symmetric situation, reduction in θ_{ij} leads to greater welfare gains in country j than in country i.

[10] To be precise, changes in wages are changes in country i's *relative* wages since country j's wages are normalized to unity.

Lemma 1 *Unilateral trade liberalization has the following effects on the equilibrium variables:*

(i) *The wage rate falls in a liberalizing country.*

(ii) *The domestic productivity cutoff rises in both liberalizing and non-liberalizing countries. As a result, the domestic trade share falls in both countries.*

(iii) *Unilateral trade liberalization is always welfare-enhancing for both countries. Starting from a symmetric situation, the welfare effect is always greater in a liberalizing country than in a non-liberalizing country.*

Lemma 1 is essentially the same as the key result in Demidova and Rodríguez-Clare (2013).[11] They find, without resorting to a specific productivity distribution and an outside good, that endogenous wages reverse the impact of asymmetric trade liberalization on welfare in a liberalizing country due to a failure of the home market effect on trade patterns. One of the crucial differences is that they graphically show their result with a simple figure, while we analytically show our result with the hat algebra. More important is our tractability in studying the impact of another competitive measure, market size, which is examined in a parallel manner with trade liberalization without using a specific productivity distribution and and outside good (Section 3.3). Further, our analytical solutions of comparative statics allow us to address qualitative and quantitative effects of unilateral changes in these competitive pressures on optimal trade policy (Sections 4 and 5).

3.3 Market Size

We next consider the impact of market size, holding other exogenous variables constant, which also has been extensively explored in the literature. Melitz and Ottaviano (2008) are the first to show that a country with larger size entails higher productivity and larger welfare through tougher competition in a domestic market. Owning to the presence of an outside good that leads to fixed wages, they find that trade liberalization and market size have an opposite impact on welfare: a unilaterally liberalizing country is worse off by relocating entry from a liberalizing country to a non-liberalizing country, as in Section 3.2. In this section, we show that, in the absence of an outside good, the impact of market size is reversed: a unilaterally expanding country is worse off by relocating entry from an expanding country to a non-expanding country. The result implies that inefficient firms survive in a domestic market with large size, which we refer to as "weak domestic selection." Although this channel via selection negatively affects productivity and welfare, a large country can nonetheless enjoy welfare gains from its market size because a negative effect on weak domestic selection may be dominated by a positive effect on increased product variety.

Just like Section 3.2, we can extract the impact of unilateral market expansion on equilibrium by keeping variable trade costs constant. Solving (5), (6) and (7) by setting $\hat{\theta}_{ji} = 0$ for the eight unknowns ($\hat{\varphi}^*_{ii}, \hat{\varphi}^*_{ij}, \hat{B}_i, \hat{w}_i$ for i, j), we get the following equilibrium relationships:

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\rho(\alpha_j + 1)}{\Xi}\hat{L}_i, \\ \hat{\varphi}^*_{jj} &= \frac{\rho(\alpha_i + 1)}{\Xi}\hat{L}_i, \\ \hat{w}_i &= \frac{\rho^2(\alpha_i\alpha_j - 1)}{\Xi}\hat{L}_i. \end{aligned} \tag{11}$$

[11] The result also relates to Felbermayr et al. (2013), though their analysis is less general than ours, in the sense that it relies on a Pareto distribution. The restriction is shown to have important consequences for policy implications.

(11) shows that expansion in L_i decreases φ_{ii}^* but increases φ_{jj}^* and w_i. The impact of market size is different from that of trade liberalization in (9), though both shocks induce tougher competition in country i's market. From (8), it then follows that welfare always rises in country j, but can rise or fall in country i depending on the extent to which expansion in market size decreases the domestic productivity cutoff there.

Intuition is again clearly explained by solving (5) and (6) first without (7):

$$\begin{aligned}\hat{\varphi}_{ii}^* &= -\frac{\alpha_j + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i,\\ \hat{\varphi}_{jj}^* &= \frac{\alpha_i + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i.\end{aligned} \tag{12}$$

Comparison between (10) and (12) immediately reveals that the direct effect of market size is absent in (12) due to the peculiar and restrictive property of CES preferences and monopolistic competition, and there is only the indirect effect through changes in wages. Thus, if wages are exogenously fixed by an outside good, market size has no impact on the domestic productivity cutoff. As expansion in market size does not directly induce entry or exit, (8) implies that unilateral market expansion increases welfare in an expanding country due solely to increased product variety, which is quite standard in a heterogeneous firm model (Melitz, 2003), as well as in a homogeneous firm model (Krugman, 1980).

If wages are endogenous, expansion in L_i leads to higher wages in country i when firms incur trade costs. The indirect effect decreases expected profits in an expanding country by raising production costs of firms, but the opposite is true in a non-expanding country. Hence, increase in w_i (by expansion in L_i) deters entry in country i but induces entry in country j. As a result, the domestic trade share rises in country i but falls in country j. Note that the negative effect on φ_{ii}^* follows from Krugman (1980)'s home market effect on wages, although the negative effect is absent in his model where productivity is exogenous. In our model, however, higher wages cause higher marginal costs and lower profitability in an expanding country. This relocates entry from an expanding country with higher wages to a non-expanding country with lower wages, which allows inefficient firms to survive in a domestic market. The intuition explains why market expansion in one country simultaneously affects productivity in another country, which does not arise when wages are exogenously fixed (Melitz and Ottaviano, 2008).[12]

What about the effect of L_i on welfare in an expanding country? Inspection of (8) shows that it depends on the magnitude of reduction in φ_{ii}^* and expansion in L_i, which capture the effects of weak domestic selection and increased product variety, respectively, affecting welfare oppositely. To see which one dominates in equilibrium, express changes in welfare in terms of changes in φ_{ii}^* only (see Appendix A.7):

$$\hat{W}_i = \frac{1}{\sigma - 1}\left((\sigma - 1)(\beta_i + \rho) - \frac{\sigma\beta_i}{\mu_i} - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\right)\hat{\varphi}_{ii}^*. \tag{13}$$

Since expansion in L_i decreases φ_{ii}^*, (13) means that country i benefits from market expansion if the value in the brackets is negative. Unfortunately, this is not always the case, and we cannot say for sure that unilateral market expansion generates welfare gains in our model. It is possible to show, however, that starting from a symmetric situation ($\alpha_i = \alpha_j$, $\beta_i = \beta_j$) and free trade ($\mu_i = 1$), such expansion unambiguously improves welfare in both expanding and non-expanding countries.

[12] Like unilateral trade liberalization, unilateral market expansion does not generate the home market effect on trade patterns when wages are endogenous. In the working paper version of this paper (Ara, 2021), we show that market expansion in country i changes the trade patterns in favor of country j through both the intensive and extensive margins. This is similar to the finding by Bertoletti and Etro (2017) who show that market expansion leads to the shift in trade patterns regarded as business destruction: an expanding country with high wages is characterized by concentration of exporting firms.

Lemma 2 *Unilateral market expansion has the following effects on the equilibrium variables:*

(i) *The wage rate rises in an expanding country.*

(ii) *The domestic productivity cutoff falls in an expanding country but rises in a non-expanding country. As a result, the domestic trade share rises in an expanding country but falls in a non-expanding country.*

(iii) *Starting from a symmetric situation and free trade, unilateral market expansion is welfare-enhancing for both expanding and non-expanding countries.*

The result in Lemma 2 has a noticeable difference from that in previous studies. In their influential work on allocation efficiency under general preferences, Dhingra and Morrow (2019) find that market expansion provides welfare gains when consumer preferences are "aligned," i.e., when demand shifts alter private and social markups in the same directions. Their result implies that market expansion improves welfare under CES preferences (since markups are invariant to market size), but it is not true in the present model. As shown by Dhingra and Morrow (2019), one of the sufficient conditions for welfare gains is that market expansion does not have a negative impact on productivity. This condition is not satisfied here because market expansion leads to weak domestic selection that negatively affects productivity in an expanding country. Hence, market expansion does not always lead to welfare gains due to distortions from weak domestic selection in our setting, whereas distortions stem from variable markups in their setting.[13]

4 Trade Policy

So far, we have examined the impact of exogenous changes in competitive measures on key endogenous variables without specifying a productivity distribution and relying on an outside good. In this section, we show that the generality is important for the characterization of a country's optimal trade policy.

4.1 Optimal Tariffs

Suppose that a government sets the tariff rate to maximize welfare. Below we mainly focus on characterizing optimal tariffs for country i, i.e., the tariffs country i would impose without fearing retaliation from country j. Appendix B provides the analysis of Nash tariffs, i.e., the tariffs each country would impose by taking account of retaliation from another country.

Consider the effect of country i's tariffs τ_{ji}, holding all other exogenous variables (including country j's tariffs τ_{ij}) constant. In country j, the effect of τ_{ji} is essentially the same as that of variable trade costs θ_{ji}, in the sense that changes in welfare are uniquely determined by changes in the domestic productivity cutoff φ^*_{jj}. From Lemma 1, it follows that increase in τ_{ji} decreases the cutoff and therefore worsens country j's welfare. In country i, in contrast, there is an additional effect of τ_{ji} on welfare: increase in τ_{ji} improves the terms of trade for country i, which operates through changes in the tariff multiplier μ_i. Because of this new channel, changes in country i's welfare corresponding to (8) are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii} + \frac{\lambda_{ji}}{\rho} \hat{\tau}_{ji}.$$

[13] Felbermayr and Jung (2018) also show that a larger country tends to have weaker domestic selection; however, their analysis of market size is confined to a Pareto distribution.

The first term is a welfare loss from tariffs due to protection of inefficient firms from foreign competition, and the second term is a welfare gain from tariffs due to improvement in the terms of trade. Upon rearrangement, welfare changes associated with these two effects are captured solely by changes in φ_{ii}^* (see Appendix A.8):

$$\hat{W}_i = \frac{\lambda_{ji}(\beta_i - \rho\alpha_i)}{\rho} \left(\frac{\beta_j - \rho\alpha_j}{\beta_j + \rho} - \frac{1}{\tau_{ji}} \right) \hat{\varphi}_{ii}^*. \tag{14}$$

Let us first consider the effect of tariffs on country i's welfare in the neighborhood of free trade at $\tau_{ji} = 1$. Recall from Lemma 1 that increase in τ_{ji} also decreases φ_{ii}^*. Setting $\tau_{ji} = 1$ in (14) immediately reveals that small increase in tariffs τ_{ji} from free trade unambiguously improves country i's welfare (which comes at the expense of country j) and the welfare-maximizing optimal tariffs are strictly positive for country i. It is also possible to show that, starting from a symmetric situation, country i's gain from tariffs cannot compensate for country j's loss, and hence the effect of τ_{ji} on world welfare is always negative.

Before moving to characterizing country i's optimal tariffs, it is useful to relate the expression in (14) with that in existing quantitative models in the literature. Using λ_{ii} and μ_i in terms of α_i defined in Section 3.1, we can alternatively express (14) as

$$\hat{W}_i = -\frac{\alpha_i}{\beta_i}\hat{\lambda}_{ii} + \left(\frac{\beta_i - \rho\alpha_i}{\rho\beta_i} \right) \hat{\mu}_i. \tag{15}$$

Welfare changes in (15) encompass the results in Arkolakis et al. (2012) without tariff revenues and those in Felbermayr et al. (2015) with tariff revenues for the Melitz (2003) model under a Pareto distribution, which is by far one of the most commonly used distributions in the literature. While this distributional assumption is known to provide a reasonable approximation for the firm size distribution, it entails some specific limitations. In particular, when productivity is Pareto distributed with a shape parameter k, the extensive margin elasticity is constant at $\gamma_{ii} = \gamma_{ij} \equiv \gamma = k - (\sigma - 1)$, meaning that the effect of trade costs on firm entry and exit is of the same magnitude between domestic and foreign markets. Moreover, substituting γ_{ii}, γ_{ij} into β_i introduced in Section 3.1, we find that β_i/α_i equals $\sigma - 1 + \gamma_{ij}$, i.e., the trade elasticity initially shown by Chaney (2008) under a Pareto distribution. As the extensive margin elasticity is constant, the trade elasticity is also constant across different markets. Denoting this unique trade elasticity by $\varepsilon \equiv \sigma - 1 + \gamma$, (15) is expressed as

$$\hat{W}_i = -\frac{1}{\varepsilon}\hat{\lambda}_{ii} + \left(1 + \frac{\eta}{\varepsilon} \right) \hat{\mu}_i,$$

where $\eta \equiv \frac{k}{\sigma-1}(1 + \frac{1-\sigma}{k}) > 0$. The above expression shows that welfare changes are captured solely by the two sufficient statistics λ_{ii} and ε without import tariffs as indicated by the first term (Arkolakis et al., 2012), but their welfare formula requires qualification with import tariffs that raise government revenues as indicated by the second term (Felbermayr et al., 2015).

The results however depend critically on the assumption that the trade elasticity is unique across markets, as emphasized by Melitz and Redding (2015). From the definition of $\gamma_{ij} \equiv -d \ln \int_{\varphi_{ij}^*}^{\varphi_{\max}} \varphi^{\sigma-1} dG_i(\varphi) / d \ln \varphi_{ij}^*$, the extensive margin elasticity generally differs across markets when a productivity distribution is unrestricted, in which case the trade elasticity is bilateral-specific to country-pairs i, j. Denoting this variable trade elasticity by $\varepsilon_{ij} \equiv \sigma - 1 + \gamma_{ij}$, (15) is further expressed as

$$\hat{W}_i = \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \left(\hat{M}_i^e - \hat{\lambda}_{ii} \right) + \left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \right) \hat{\mu}_i. \tag{16}$$

This expression is a counterpart to that in Melitz and Redding (2015, equation (33)), though we derive welfare changes by tariffs that raise government revenues. Note that, beside the domestic trade share λ_{ii} and the trade elasticity ε_{ij}, welfare changes also depend on the extensive margin elasticity differential between domestic and export markets $\gamma_{ii} - \gamma_{ij}$, which arises whenever the trade elasticity differs across markets. They argue that, when there exists the differential, the domestic trade share and the trade elasticity are no longer sufficient statistics for welfare, and welfare changes can be substantially mis-estimated if the trade elasticity is assumed constant despite that the "true" elasticity is variable. (16) shows that this critique applies to welfare changes associated with tariffs. For example, when the extensive margin is more elastic in an export market than in a domestic market ($\gamma_{ii} - \gamma_{ij} < 0$), welfare changes are smaller than those without this differential ($\gamma_{ii} - \gamma_{ij} = 0$). Recent empirical work documents that the trade elasticity indeed substantially differs across country-pairs, supporting their welfare result.[14]

We turn to characterizing the optimal tariffs for country i. As in most of previous work in the trade policy literature, we use the first-order condition of welfare maximization by assuming the sufficiency to be satisfied. Then setting $\hat{W}_i = 0$ in (14) and solving for τ_{ji} yields the following expression for optimal tariffs:

$$\tau_{ji}^* = 1 + \underbrace{\frac{\rho}{\frac{\alpha_j}{\alpha_j+1}\left(\frac{\beta_j}{\alpha_j} - \rho\right)}}_{t_{ji}^*} = \frac{\beta_j + \rho}{\beta_j - \rho\alpha_j} > 1.$$

Moreover, noting that $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ and rewriting the definition of β_j in terms of the trade elasticity ε_{ji}, we find that the optimal tariffs are implicitly characterized as a function of key observable moments:

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}\left(\varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})(1 - \tilde{\lambda}_{jj}) - \rho\right)}. \tag{17}$$

(17) shows that the optimal tariffs for country i are inversely related to country j's export supply elasticity, which is composed of both the domestic trade share $\tilde{\lambda}_{jj}$ and the trade elasticity ε_{ji}, as in existing trade models. One of the crucial differences in this model, however, is that the extensive margin elasticity differential $\gamma_{jj} - \gamma_{ji}$ enters the expression of the optimal tariffs, which reflects the aspect that the trade elasticity is not necessarily unique across different markets.

The optimal tariff formula (17) can be regarded as a generalization of some of the well-known results in the trade policy literature. If productivity is Pareto distributed with a shape parameter k, the extensive margin elasticity is constant at $\gamma_{jj} = \gamma_{ji} = \gamma = k - (\sigma - 1)$ and the trade elasticity is constant at $\varepsilon_{ji} = \varepsilon = \sigma - 1 + \gamma$ as shown above. Since $\varepsilon = k$ in that case, (17) reduces to

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}(k - \rho)}. \tag{18}$$

This expression is exactly the same as the optimal tariffs shown by Felbermayr et al. (2013) in a heterogeneous firm model à la Melitz (2003) in which firms draw productivity from a Pareto distribution. Furthermore, it is also possible to consider a homogeneous firm model as a special case of a heterogeneous firm model in which firms draw productivity of either zero or constant from a degenerated distribution (Melitz and Redding, 2015).

[14]Maintaining CES preferences and monopolistic competition so that the intensive margin is constant, Helpman et al. (2008) find that there is substantial variation in the trade elasticity across country-pairs due to the extensive margin. In a similar vein, Bas et al. (2017) show that the extensive margin varying with country-pairs plays a key role in quantifying the trade elasticity. These pieces of evidence suggest the existence of the extensive margin elasticity differential.

If trade costs are sufficiently low so that all homogeneous firms export in this class of the model, we can easily show that the extensive margin elasticity is constant at $\gamma_{jj} = \gamma_{ji} = \gamma = 0$ and the trade elasticity is constant at $\varepsilon_{ji} = \varepsilon = \sigma - 1$. In that case, thus, (17) reduces to

$$t^*_{ji} = \frac{1}{\tilde{\lambda}_{jj}(\sigma - 1)}. \tag{19}$$

This expression is exactly the same as the optimal tariffs shown by Gros (1987) in a homogeneous firm model à la Krugman (1980).

At this standpoint, the optimal tariff formula (17) poses two caveats. First, we cannot always say that the optimal tariffs are smaller in the heterogeneous firm model than in the homogeneous firm model. Just as the different trade models yield the different domestic trade shares $\tilde{\lambda}_{jj}$, these models also yield the different trade elasticities ε_{ji}. This means that the optimal tariffs in the different trade models are not directly comparable without controlling for the difference in the trade elasticity. Our formula is useful for shedding light on this point. Plugging (17) in $\gamma_{jj} - \gamma_{ji} = 0$ that holds in the heterogeneous model with a Pareto distribution and the homogeneous firm model with a degenerated distribution, we find that conditional on the two empirically observable moments above, the optimal tariffs are the same between the different trade models. The result is, of course, obtained by applying the welfare formula by Arkolakis et al. (2012) to our optimal tariff formula: conditional on the two sufficient statistics for welfare, changes in welfare associated with tariffs are the same; and, consequently, levels of the optimal tariffs are also the same.

Second, the equivalence of the optimal tariffs across the different trade models holds only if the extensive margin elasticity differential is zero ($\gamma_{jj} - \gamma_{ji} = 0$). If the condition is violated, however, the optimal tariffs are different even after controlling for the two sufficient statistics for welfare. Consider, for example, the case in which the extensive margin is more elastic in an export market than in a domestic market ($\gamma_{jj} - \gamma_{ji} < 0$). As seen in (16), welfare changes associated with tariffs are smaller than those without the differential. Since the welfare-maximizing optimal tariffs are strictly positive, this implies in the trade policy context that the government faces a smaller welfare loss from tariffs and has a more incentive to impose higher tariffs. Indeed, (17) shows that levels of the optimal tariffs are higher for $\gamma_{jj} - \gamma_{ji} < 0$ than for $\gamma_{jj} - \gamma_{ji} = 0$, conditional on the two sufficient statistics. This arises because the government does not take account of the difference in the impact of tariffs on firm entry and exit across markets. The converse is true for another case ($\gamma_{jj} - \gamma_{ji} > 0$) and levels of the optimal tariffs are lower than those in the absence of this differential. In general cases, thus, the domestic trade share and the trade elasticity are no longer sufficient statistics not only for welfare as in Melitz and Redding (2015), but also for optimal trade policy.

Proposition 1 *Conditional on the domestic trade share and the trade elasticity, levels of the optimal tariffs have the following properties:*

(*i*) *When the extensive margin elasticity is the same between domestic and export markets, levels of the optimal tariffs are the same across the different trade models.*

(*ii*) *When the extensive margin is more (less) elastic in an export market than in a domestic market, levels of the optimal tariffs are higher (lower) than those in the absence of this differential.*

In Proposition 1, we compare the optimal tariffs across the different trade models, holding *both* the domestic trade share and the trade elasticity equal that endogenously arise in the respective model. If the optimal tariffs are compared without such conditioning, the proposition no longer holds. The optimal tariffs in (17), (18) and

(19) depend on the domestic trade share, which is a function of tariffs and hence is not always the same level. The fact that the optimal tariffs are implicitly characterized means that we cannot solve for the optimal tariffs in closed forms as in existing work (Gros, 1987; Felbermayr et al., 2013). To avoid this difficulty, Felbermayr et al. (2013) compare the optimal tariffs in the heterogeneous firm model and the homogeneous firm model, holding *only* the domestic trade share equal. Recently, Costinot et al. (2020) show that the optimal tariffs can be lowered under a non-Pareto distribution (relative to those under a Pareto distribution). Although they also stress the role of a general productivity distribution in characterizing the optimal tariffs as in our paper, the optimal tariffs are compared under the same condition as that in Felbermayr et al. (2013). Unfortunately, we cannot adopt their conditioning because not only is the domestic trade share but also the trade elasticity and the extensive margin elasticity differential are a function of tariffs. Thus we cannot see which optimal tariffs are lowest among (17), (18) and (19) without conditioning on equilibrium variables of the different models. For this reason, we use numerical solutions in Section 5 to figure out whether variability of the trade elasticity causes a quantitatively significant discrepancy in levels of the optimal tariffs.

4.2 Comparative Statics

Let us examine the effects of competition on the optimal tariffs. Consider the optimal tariffs with $\gamma_{jj} - \gamma_{ji} = 0$ in (18) and (19). In this case, changes in exogenous variables affect the optimal tariffs τ^*_{ji} only through the domestic trade share $\tilde{\lambda}_{jj}$. Lemma 1 says that reduction in any trade costs increases the domestic productivity cutoff φ^*_{jj} which decreases the domestic trade share $\tilde{\lambda}_{jj}$. Lemma 2 also says that expansion in country i's size increases φ^*_{jj} which decreases $\tilde{\lambda}_{jj}$. From these comparative statics results, it follows that country i's optimal tariffs are higher, the lower the trade costs between two countries or the larger the market size in country i. These properties of optimal tariffs are consistent with those of optimal tariffs with a constant trade elasticity in the literature (Gros, 1987; Felbermayr et al., 2013).

Next, consider the optimal tariffs with $\gamma_{jj} - \gamma_{ji} \neq 0$ in (17). In this case, changes in exogenous variables affect the optimal tariffs not only through the domestic trade share $\tilde{\lambda}_{jj}$ but also through the trade elasticity ε_{ji}. This additional channel through endogenous changes in the trade elasticity can be shown more formally by making clear the relationship between the extensive margin elasticity differential and the trade elasticity. From the comparative statics results in Lemmas 1 and 2, we find that when $\gamma_{jj} - \gamma_{ji} \neq 0$, the trade elasticity is no longer constant and thus endogenously responds to exogenous shocks. In case of variable trade costs θ_{ji}, for example, we have the following relationship (see Appendix A.9):

$$\gamma_{jj} - \gamma_{ji} \lesseqgtr 0 \implies \frac{d\varepsilon_{ji}}{d\theta_{ji}} \gtreqless 0.^{15} \tag{20}$$

(20) shows that when the extensive margin is more elastic in an export market than in a domestic market so that the differential is negative, reduction in variable trade costs decreases the trade elasticity. This accords well with recent evidence that the trade elasticity is small for proximate country-pairs where the trade volume is already large (Bas et al., 2017). When the differential is positive, such reduction increases the trade elasticity. These endogenous changes in the trade elasticity imply that, when that elasticity differs across markets, the aforementioned properties of optimal tariffs are not always satisfied. Only when the extensive margin is unique across markets so that the differential is zero, is the trade elasticity constant and hence invariant to changes in variable trade costs.

[15] Strictly speaking, we need to assume a productivity distribution such that γ_{jn} is a monotonic function in φ^*_{jn} for this result, so that the sign of $\gamma_{ji} - \gamma_{jj}$ does not switch with changes in exogenous shocks (as long as selection into exporting is satisfied).

We can show that the properties of optimal tariffs are critically affected by exogenous shocks in our model. Consider reduction in variable trade costs θ_{ji}. When the differential is negative ($\gamma_{jj} - \gamma_{ji} < 0$), such reduction decreases the trade elasticity ($\frac{d\varepsilon_{ji}}{d\theta_{ji}} > 0$) as well as the domestic trade share in country j ($\frac{d\tilde{\lambda}_{jj}}{d\theta_{ji}} > 0$). Due to an extra adjustment through ε_{ji} that is absent in the optimal tariffs in (18) and (19), the impact of variable trade costs on the optimal tariffs in (17) is reinforced. When the differential is positive, the converse is true, in that the impact on the optimal tariffs is attenuated. Only when there is no differential, is the trade elasticity constant and reduction in θ_{ji} affects the optimal tariffs only through decreases in the domestic trade share. These highlight a possibility that the effect of variable trade costs on the optimal tariffs can be substantially mis-estimated when the trade elasticity is assumed constant despite that the "true" trade elasticity is variable. In other words, there can be a discrepancy in the optimal tariffs not only in terms of levels but also in terms of changes associated with exogenous shocks (see Appendix A.10).

Proposition 2 *Reduction in trade costs between the two countries and expansion in country i's size lead to the following changes in the optimal tariffs for country i:*

(*i*) *When the extensive margin elasticity is the same between domestic and export markets, they increase the optimal tariffs only through decreases in the domestic trade share.*

(*ii*) *When the extensive margin is more (less) elastic in an export market than in a domestic market, they reinforce (attenuate) the impact on the optimal tariffs by decreasing (increasing) the trade elasticity.*

One of the interesting results in Proposition 2 arises when the extensive margin is less elastic in an export market than in a domestic market ($\gamma_{jj} - \gamma_{ji} > 0$). In this case, the model predicts that the optimal tariffs for country i are lower, the lower are trade costs and the larger is country i's size. From a policy point of view, the effect of market size is of particular interest. If a government in a large country chooses the tariff rate, it can enjoy a terms-of-trade gain by setting tariffs, just like the conventional optimal tariff theory. However, in the presence of firm heterogeneity, a large country suffers from weak domestic selection which negatively affects welfare by allowing inefficient firms to survive there. With this selection effect, the imposition of tariffs accelerates the welfare loss from protecting inefficient firms against foreign competition. Taken together, the optimal tariffs are decreasing in market size only if the welfare loss from protecting inefficient firms by tariffs is stronger than the welfare gain from improving the terms-of-trade by tariffs, which occurs under the condition that $\gamma_{jj} - \gamma_{ji} > 0$ in this model. To the best of our knowledge, however, there is no empirical evidence that supports $\gamma_{jj} - \gamma_{ji} > 0$. Therefore, in the next section, we demonstrate that, even when $\gamma_{jj} - \gamma_{ji} < 0$ so that the optimal tariffs are strictly increasing in market size as in existing work, its impact on the optimal tariffs is quantitatively very limited (relative to that of variable trade costs). These results highlight one of the main policy implications from our analysis that holds regardless of the sign of differential: when the trade elasticity differs across markets, a large country would not necessarily enjoy large welfare gains from setting high tariffs.

5 Quantitative Relevance

This section explores the quantitative relevance of our theoretical results. Using standard values of the model's parameters in analytical solutions, we numerically compare the optimal tariffs across the different trade models. Appendix C offers a detailed discussion of procedures and parameter values in the quantitative exercise below, drawing on the working paper version of this paper (Ara, 2021).

5.1 Calibration

To introduce a variable trade elasticity in the analysis, we employ a *bounded* Pareto distribution. Specifically, when productivity φ is Pareto distributed with a shape parameter k with support $(\varphi_{\min}, \varphi_{\max})$, the distribution is given by the following functional form (Feenstra, 2017):

$$G_i(\varphi) = \frac{1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k}{1 - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

Notice that when the upper bound is infinite ($\varphi_{\max} = \infty$), this collapses to an *unbounded* Pareto distribution that is often used in the literature, in which case the extensive margin elasticity is the same across markets. However, when the upper bound is finite ($\varphi_{\max} < \infty$), the extensive margin elasticity differs across markets.[16] Thus, the latter is apt for (17) while the former is apt for (18) or (19). Specifying the productivity distribution and using values of the parameters from the existing literature, we are able to uniquely determine values of the domestic and export productivity cutoffs. These values in turn pin down values of the three key moments of optimal tariffs $\varepsilon_{ji}, \gamma_{jj} - \gamma_{ji}, \tilde{\lambda}_{jj}$ that appear in (17), (18) and (19).

Our interest is in addressing how the optimal tariffs with a variable trade elasticity, (17), are quantitatively different from those with a constant trade elasticity, (18) or (19), for given levels of exogenous variables. The problem is that all of the optimal tariffs depend on the domestic trade share which is a function of tariffs, and hence we cannot directly compare them without conditioning on some equilibrium variables. For this reason, we compare (17), (18) and (19) holding the values of productivity cutoffs equal across the different models in an initial equilibrium. With such conditioning, numerical solutions greatly help make the comparison.

Using the analytical solutions of comparative statics results in Section 3, we are also able to investigate the quantitative impacts of exogenous shocks on the optimal tariffs. For simplicity, we assume that country i unilaterally changes variable trade costs θ_{ji} and market size L_i in order to explore their impacts on country i's optimal tariffs t^*_{ji}. Proposition 2 suggests that reduction in θ_{ji} and expansion in L_i have *qualitatively* similar effects on t^*_{ji}. Nevertheless, the numerical exercise allows us to address how the two exogenous changes have *quantitatively* different effects on t^*_{ji} from an initial equilibrium.

The formula in (17), (18) and (19) holds for the optimal tariffs set by country i on imports from country j, requiring the key moments in country j. This means that, when choosing standard values of the parameters based on estimates from US data, we need to treat the United States as country j in the numerical exercise. In other words, the optimal tariffs we quantify are those *faced* by the United States. We do not try to quantify the optimal tariffs *chosen* by the United States, as the parameter values of other countries are hard to find in the existing empirical literature relative to those of the United States. We follow Felbermayr et al. (2013) in assuming that the two countries differ in their tariff rate but are otherwise identical in an initial equilibrium. For simplicity, we treat country i as the rest of world and consider a situation where country i optimally sets the tariff rate taking country j's tariff rate as given.

5.2 Results

Figure 2 shows the quantitative comparison of the optimal tariffs across the different trade models. Panel A is case of variable trade costs, while Panel B is case of market size. In both panels, the solid, dashed and dotted

[16] This distribution is employed by Helpman et al. (2008) to develop a gravity equation model with a variable trade elasticity. Using a log-normal distribution, Head et al. (2014) also show that the trade elasticity is variable whereby the extensive margin is more elastic in an export market than in a domestic market. In both cases, the differential in (20) is negative.

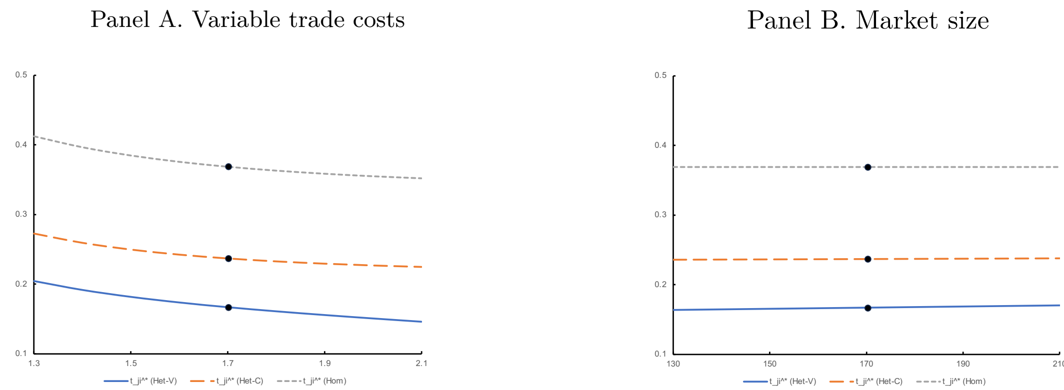

Figure 2: Optimal tariffs across different trade models

Note: In an initial equilibrium, $\theta_{ij} = \theta_{ji} = 1.7$ and $L_j = L_j = 170$. See Appendix C for a discussion of other parameter values.

curves represent the optimal tariffs in (17), (18) and (19), where the dots denote the optimal tariffs in an initial equilibrium which are 16.6 percent, 23.6 percent and 36.7 percent, respectively. The numerical comparison indicates that a variable trade elasticity lowers the optimal tariffs substantially: levels of the optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model. The results illustrate the quantitative relevance of Proposition 1: levels of the optimal tariffs are quantitatively quite different across the different trade models, conditional on key equilibrium variables.

To understand the point, compare the optimal tariffs in (18) and (19). In estimating (19), we consider an extended homogeneous firm model where firms draw productivity of either zero or constant from a degenerated distribution (Melitz and Redding, 2015). As values of the productivity cutoffs are equal across (18) and (19), values of all the equilibrium variables (including the probability of entry and exporting) are also equal so that the two models generate the same aggregate variables (including the domestic trade share $\tilde{\lambda}_{jj}$) in an initial equilibrium. Consequently, the only difference between (18) and (19) is the structural parameter values, which implies that the optimal tariffs are lower in the heterogeneous firm model than in the homogeneous firm model, holding the domestic trade share equal. In fact, levels of the optimal tariffs in our numerical exercise are of comparable magnitude to those in the existing literature.[17]

Compare the optimal tariffs in (17) and (18). In estimating (17), we consider a finite upper bound so that the extensive margin elasticity differs across markets. Since this elasticity is the same across markets in (18), the key difference is whether the extensive margin elasticity is variable or not, which has two critical effects on optimal tariffs. First, the trade elasticity $\varepsilon_{ji} = \sigma - 1 + \gamma_{ji}$ is endogenously greater when the upper bound is finite, lowering (17) relative to (18). Second, the extensive margin elasticity differential $\gamma_{jj} - \gamma_{ji}$ is negative when the upper bound is finite, raising (17) relative to (18), as seen in Section 4.1. In our numerical exercise where the domestic trade share $\tilde{\lambda}_{jj}$ is large enough, (17) implies that the latter is dominated by the former. Hence the optimal tariffs are lower in the heterogeneous firm model with a variable trade elasticity than in that with a constant trade elasticity.

Next, we report the quantitative impacts of variable trade costs and market size on the optimal tariffs. Our analytical solutions of comparative statics reveal that these two exogenous variables have different effects

[17] Felbermayr et al. (2013) find that optimal tariffs are 26.4 percent in the heterogeneous firm model with a Pareto distribution. Levels of the optimal tariffs are not the same as theirs, because we choose the parameter values in Melitz and Redding (2015).

on productivity cutoffs. Exploiting the model's outcome, we can address how unilateral changes in θ_{ji} and L_i have quantitatively different effects on the optimal tariffs. For expositional purposes, we restrict attention here to changes in (17). We find, for example, that 17.6 percent reduction in variable trade costs (from $\theta_{ji} = 1.7$ to $\theta_{ji} = 1.4$) increases country i's optimal tariffs by 14.7 percent (from $t^*_{ji} = 0.166$ to $t^*_{ji} = 0.191$). However, 17.6 percent expansion in market size (from $L_i = 170$ to $L_i = 200$) increases the optimal tariffs by 1.5 percent (from $t^*_{ji} = 0.166$ to $t^*_{ji} = 0.169$). Thus, the effect of variable trade costs is nearly ten times as large as that of market size. The results illustrate the quantitative implications of Proposition 2: the impact of exogenous variables on the optimal tariffs is quantitatively quite different.

Intuition behind the results is explained by the fact that market size has no direct effect on productivity cutoffs under CES preferences and monopolistic competition. (10) shows that variable trade costs have both the direct effect and the indirect effect through changes in wages; however, (12) shows that market size has the indirect effect only. In addition, our analytical solutions of comparative statics results show that the indirect effect is of the same magnitude between (10) and (12) in an initial equilibrium. Then, it follows that unilateral changes in variable trade costs have a larger effect on the optimal tariffs than those in market size, due to the direct effect that is missing in (12).

The quantitative exercise confirms our policy implications. Although a large country benefits from terms-of-trade improvement, it suffers from weak domestic selection whereby tariffs exacerbate this selection effect even further by protecting inefficient firms. In our calibration, the optimal tariffs are increasing in market size (as shown in Figure 2), which reflects that the benefit of tariffs is greater than the cost of tariffs and therefore the country enjoys welfare gains from tariffs. However, the government needs to take into account not only the gains from terms-of-trade but also the losses of weak domestic selection, attributing to a limited effect on the optimal tariffs and welfare gains. Our calibration also has the practical relevance to real-world trade policies. Consider recent tariff hikes under the Trump administration. While the optimal tariffs are those faced by the United States in our calibration, the quantitative nature of our results would hold for the US optimal tariffs. Thus, the exercise implies that the US optimal tariffs would be potentially smaller than the observed tariffs in recent tariff hikes which would have only limited effects on US welfare, if any.

5.3 Role of Generality

Now we are able to explain the role of our generality in deriving the policy implications. In the Introduction, we noted that our model has the three distinctive features: (*i*) the trade elasticity differs across markets; (*ii*) the wage rate is endogenous; and (*iii*) the government sets the tariff rate. Clearly, if we drop (*iii*), the optimal tariffs cannot be derived, implying that nuanced policy implications in this paper come from (*i*) and (*ii*).[18] If we drop (*i*), levels of the optimal tariffs are higher, as shown in Figure 2. Changes in the optimal tariffs are also critically affected by (*i*). While our analytical solutions show that all of our results hold without (*i*), the impact of exogenous variables on the optimal tariffs is much stronger with (*i*). For example, we find that changes from $\theta_{ji} = 1.7$ to $\theta_{ji} = 1.4$ increase the optimal tariffs by 9.4 percent in (18), which is smaller than 14.7 percent in (17), as seen above. This reflects Proposition 2: the trade elasticity endogenously responds to any exogenous shocks in (17), whereas the elasticity is constant in (18). On the other hand, if we drop (*ii*), the indirect effect through changes in wages disappears (see (10) and (12)), altering the quantitative impacts on the optimal tariffs. In a nutshell, our generality is useful in quantifying the optimal tariffs in terms of both levels and changes.

[18] As pointed out by an anonymous referee, the government might set other import barriers. The optimal level of such barriers would satisfy the properties in Propositions 1 and 2, but we focus on import tariffs to stress the novelty of our optimal tariffs.

6 Conclusion

This paper presents a heterogeneous firm model of trade to study optimal tariffs with a variable trade elasticity. To provide a better understanding of the effects of trade liberalization and market size on optimal trade policy, we consider a general setting where the trade elasticity is bilateral-specific to country-pairs and the wage rate is endogenously determined. Our central contributions to the literature can be summarized as follows. Optimal levels of import tariffs are inversely related to the two empirically observable moments—the domestic trade share and the trade elasticity—where the second integrant is either constant or variable depending on the micro structure of the model. When the trade elasticity is constant and the same across markets as assumed before, optimal levels of import tariffs are the same between different trade models, holding both the domestic trade share and trade elasticity equal. However, when the trade elasticity is variable and differs across markets as reported by empirical work, optimal levels are mis-estimated due to the variable nature of the trade elasticity. This finding applies to changes in optimal tariffs with respect to trade liberalization and market expansion, in the sense that the effects of these two exogenous variables on optimal tariffs depend on the micro structure that makes the trade elasticity variable.

We also explore the quantitative relevance of our theoretical results. Calibrating the model to US aggregate and firm-level data, we find that levels of optimal tariffs with a variable trade elasticity are quantitatively lower than those with a constant trade elasticity (changes in optimal tariffs to exogenous shocks are quantitatively bigger with a variable trade elasticity). Our numerical solutions indeed reveal that levels of optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model, holding key endogenous variables the same across different trade models. Despite that, however, levels of optimal tariffs predicted by our model—16.6 percent—are much higher than levels of actual tariffs observed in the real world—3.2 percent—as reviewed in the Introduction. In that sense, the derivation of welfare-maximizing optimal tariffs is useful in appreciating the role played by WTO in reducing worldwide tariffs and thereby ensuring the gains from trade liberalization, even though such counterfactual non-cooperative tariffs are not permitted in reality. Hopefully, our calibration results offer practical insights for policymakers on the potential welfare consequences of real-world trade policies, such as recent tariff hikes under the Trump administration.

Nevertheless, much remains to be done. On the theory side, the variable nature of the trade elasticity comes from the extensive margin which is made possible by departing from a commonly-used Pareto distribution. However, it might as well come from the intensive margin that relates to the firm-level elasticity. To correctly examine the variability of the trade elasticity in trade policy evaluations, it is necessary to drop CES preferences with constant markups and instead adopt general preferences with variable markups that differ across firms. We expect that a variable trade elasticity would play a more critical role in optimal trade policy in that case. On the quantitative side, on the other hand, we have employed a bounded Pareto distribution to quantify levels and changes in optimal tariffs. While the distribution leads optimal tariffs to increase with market size, this may not be always true. For example, Naito (2019) finds a significantly negative relationship between GDP and tariffs across countries, suggesting that larger countries tend to impose lower tariffs. To explore the quantitative impact of such optimal tariffs, we need to replace a (bounded or unbounded) Pareto distribution with another one where the extensive margin is less elastic in an export market than in a domestic market; however, we are uncertain about which productivity distribution meaningfully yields this outcome and whether the resulting quantification is able to provide a good fit for observed aggregate and firm-level data. We leave these theoretical and quantitative extensions and their implications for optimal trade policy to future work.

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Appendices (Not for Publication unless Requested)

A Proofs

A.1 ZCP and FE conditions

We first show that the ZCP condition is given by (1). A firm with productivity φ chooses the price to maximize the profits. In monopolistic competition where firms are too small relative to the market, each firm does not affect any aggregate variables. As a firm takes P_i and R_i in consumer demand as given in choosing the price, $\frac{\partial q_{ji}(\varphi)}{\partial p_{ji}(\varphi)} = -\sigma \frac{q_{ji}(\varphi)}{p_{ji}(\varphi)}$. Solving the profit maximization problem yields the optimal price, $p_{ji}(\varphi) = \frac{\tau_{ji}\theta_{ji}w_j}{\rho\varphi}$, which generates the price-cost margin, $\frac{p_{ji}(\varphi)}{\tau_{ji}} - \frac{\theta_{ji}w_j}{\varphi} = \frac{p_{ji}(\varphi)}{\tau_{ji}\sigma}$. From $r_{ji}(\varphi) \equiv \frac{p_{ji}(\varphi)q_{ji}(\varphi)}{\tau_{ji}}$, the firm variable profits are expressed as $\frac{r_{ji}(\varphi)}{\sigma}$. Moreover, substituting the optimal price into consumer demand and rearranging,

$$r_{ji}(\varphi) = \underbrace{R_i(\rho P_i)^{\sigma-1}}_{B_i} \tau_{ji}^{-\sigma}(\theta_{ji}w_j)^{1-\sigma}\varphi^{\sigma-1}.$$

The export productivity cutoff φ^*_{ji} must satisfy $\frac{r_{ji}(\varphi^*_{ji})}{\sigma} = w_j f_{ji}$. This gives us the expression of ZCP condition in (1). Note that the domestic productivity cutoff φ^*_{jj} is obtained when $i = j$ and hence $\tau_{jj} = \theta_{jj} = 1$ in (1):

$$B_j w_j^{1-\sigma}(\varphi^*_{jj})^{\sigma-1} = \sigma w_j f_{jj}.$$

Selection into exporting requires $\varphi^*_{ji} > \varphi^*_{jj}$. Using φ^*_{ji} in (1) and φ^*_{jj} above, we get

$$\left(\frac{\varphi^*_{ji}}{\varphi^*_{jj}}\right)^{\sigma-1} = \frac{\tau_{ji}^{\sigma}\theta_{ji}^{\sigma-1}f_{ji}}{f_{jj}}\frac{B_j}{B_i}. \tag{A.1}$$

As (A.1) applies to country i by changing subscripts i, j, we find that $\varphi^*_{ij} > \varphi^*_{ii}$ and $\varphi^*_{ji} > \varphi^*_{jj}$ if and only if

$$\frac{f_{ii}}{\tau_{ij}^{\sigma}\theta_{ij}^{\sigma-1}f_{ij}} < \frac{B_i}{B_j} < \frac{\tau_{ji}^{\sigma}\theta_{ji}^{\sigma-1}f_{ji}}{f_{jj}}.$$

This requires that trade costs are sufficiently large and market size is not too different between two countries, where the latter is measured by relative market demand $\frac{B_i}{B_j}$.

We next show that the FE condition is given by (2). To derive the expected profits of firms in country i, we must calculate those earned by domestic firms and exporters. Consider, for example, the expected profits of exporters from country i to country j. With selection into exporting, only a firm with $\varphi \in (\varphi^*_{ij}, \varphi_{\max})$ earns the export profits $\frac{r_{ij}(\varphi)}{\sigma} - w_i f_{ij}$. Hence, the expected profits of exporters from country i are given by

$$\begin{aligned}\int_{\varphi^*_{ij}}^{\varphi_{\max}} \left(\frac{r_{ij}(\varphi)}{\sigma} - w_i f_{ij}\right) dG_i(\varphi) &= \int_{\varphi^*_{ij}}^{\varphi_{\max}} \left(B_j\tau_{ij}^{-\sigma}(\theta_{ij}w_i)^{1-\sigma}\varphi^{\sigma-1} - w_i f_{ij}\right) dG_i(\varphi)\\ &= w_i f_{ij}\int_{\varphi^*_{ij}}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*_{ij}}\right)^{\sigma-1} - 1\right] dG_i(\varphi),\end{aligned}$$

where the first equality comes from the variable profits ($\frac{r_{ij}(\varphi)}{\sigma}$) and the second equality comes from the export productivity cutoff (φ^*_{ij}) both derived above, albeit for exporters from country j. From the definition of $J_i(\varphi^*)$, we can express the expected profits of them as $w_i f_{ij} J_i(\varphi_{ij}^*)$. Similarly, the expected profits of domestic firms in country i are expressed as $w_i f_{ii} J_i(\varphi_{ii}^*)$. The sum of expected profits is offset by the fixed entry costs $w_i f_i^e$ under free entry so that the expected net profits are zero. This gives us the expression of FE condition in (2).

A.2 Proof of LMC and TB Conditions

We first show that the LMC condition is given by

$$L_i = \frac{R_i - T_i}{w_i}.$$

Aggregate labor in country i's economy is given by $L_i = L_i^e + L_i^p$, where L_i^e and L_i^p denote aggregate labor used for entry and production, respectively. As there are the mass M_i^e of entrants, the LMC for entry requires $L_i^e = M_i^e f_i^e$. Noting that f_i^e must satisfy the FE condition in (2) and using the definition of $J_i(\varphi^*)$,

$$L_i^e = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \frac{1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) - \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} \right\}.$$

In contrast, using $l_{ii}(\varphi) = f_{ii} + q_{ii}(\varphi)/\varphi,\ l_{ij}(\varphi) = f_{ij} + \theta_{ij} q_{ij}(\varphi)/\varphi$, the LMC condition for production requires

$$L_i^p = M_i^e \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} \left(f_{in} + \frac{\theta_{in} q_{in}(\varphi)}{\varphi} \right) dG_i(\varphi).$$

Further, noting that firm pricing rule generates the relationship $q_{ij}(\varphi) = \frac{\tau_{ij} r_{ij}(\varphi)}{p_{ij}(\varphi)} = \frac{\rho \varphi r_{ij}(\varphi)}{\theta_{ij} w_i}$,

$$L_i^p = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} + \frac{\sigma - 1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \right\}.$$

Summing up aggregate labor used for entry and production,

$$L_i = \frac{\sum_n R_{in}}{w_i},$$

where $R_{in} = M_i^e \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi)$ is aggregate revenue (or expenditure) of goods from country i to country $n = i, j$ net of tariffs. The result follows from $R_i = \sum_n \tau_{ni} R_{ni}$ and $R_{ij} = R_{ji}$.

Next, we show that the LMC condition is equivalent with the TB condition. On the one hand, aggregate labor income in country i consists of revenues by domestic firms and exporting firms of country i net of tariffs, $w_i L_i = \sum_n R_{in}$. On the other hand, aggregate expenditure in country i consists of expenditures on domestic goods and foreign goods inclusive of tariffs $R_i = \sum_n \tau_{ni} R_{ni}$. From these, the TB condition, $R_{ij} = R_{ji}$, is

$$\underbrace{R_{ii} + R_{ij}}_{w_i L_i} = \underbrace{R_{ii} + \tau_{ji} R_{ji}}_{R_i} - \underbrace{(\tau_{ji} - 1) R_{ji}}_{T_i}.$$

Hence, the TB condition is equivalent with the LMC condition, in the sense that both conditions induce the same equality, $R_i = w_i L_i + T_i$.

Finally, we show that the TB condition is given by (3). In light of $R_i = \sum_n \tau_{ni} R_{ni}$, $T_{ji} = (\tau_{ji} - 1) R_{ji}$ and $R_{ji} = R_{ij}$, we can rewrite $R_i = w_i L_i + T_i$ as $w_i L_i = R_{ii} + R_{ij}$. The result follows immediately from using $R_{ii} = \tilde{\lambda}_{ii} w_i L_i$ and $R_{ij} = \tilde{\lambda}_{ij} w_j L_j$.

A.3 Proof of Welfare

We show the derivation of (4). Welfare per worker is given by

$$
\begin{aligned}
W_i &\equiv \frac{U_i}{L_i} \\
&= \frac{R_i}{L_i P_i} \\
&= \frac{\mu_i w_i}{P_i}
\end{aligned}
$$

where the second equality follows from defining an aggregate good $Q_i \equiv U_i$ that satisfies $P_i Q_i = R_i$, and the third equality follows from noting that $R_i = \mu_i w_i L_i$ (from the definition of the tariff multiplier μ_i). Further, substituting $R_i = \mu_i w_i L_i$, aggregate market demand is expressed as

$$B_i = \mu_i w_i L_i (\rho P_i)^{\sigma-1}.$$

Substituting the above equality into the ZCP condition (1) which pins down φ_{ii}^* when $j = i$ and rearranging, the real wages in country i are expressed as

$$\frac{w_i}{P_i} = \left(\frac{\mu_i L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} \rho \varphi_{ii}^*.$$

This shows that the real wage depends not only on the domestic productivity cutoff φ_{ii}^*, but also the tariff multiplier μ_i. Reflecting that, the real wage becomes the same as that in the standard Melitz model without tariff revenues ($\mu_i = 1$) in this model; see the expression of real wage in Demidova and Rodríguez-Clare (2013). Finally, substituting w_i/P_i into above W_i establishes the result.

A.4 Proof of α_i

We first show that α_i in (6) satisfies

$$
\begin{aligned}
\alpha_i &\equiv \frac{f_{ii} J_i'(\varphi_{ii}^*)\varphi_{ii}^*}{f_{ij} J_i'(\varphi_{ij}^*)\varphi_{ij}^*} \\
&= \frac{f_{ii} (\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*)}{f_{ij} (\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)},
\end{aligned}
\tag{A.2}
$$

where $V_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \varphi^{\sigma-1} dG_i(\varphi)$ is a decreasing function of φ^*. To show the equality in (A.2), differentiating $J_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] dG_i(\varphi)$ with respect to φ^*,

$$J_i'(\varphi^*) = -\left(\frac{\sigma-1}{\varphi^*} \right) [J_i(\varphi^*) + 1 - G_i(\varphi^*)].$$

Moreover, from the functional forms of $J_i(\varphi^*)$ and $V_i(\varphi^*)$, we get the following equality:

$$J_i(\varphi^*) + 1 - G_i(\varphi^*) = (\varphi^*)^{1-\sigma} V_i(\varphi^*).$$

Finally, substituting this into $J_i'(\varphi^*)$ gives us the result.

Next, we show several properties of α_i.

- The first property is that $\alpha_i\alpha_j - 1 > 0$. To show this, it follows from (A.1) that

$$\left(\frac{\varphi_{ij}^*}{\varphi_{ii}^*}\right)^{\sigma-1} = \frac{\tau_{ij}^{\sigma}\theta_{ij}^{\sigma-1}f_{ij}}{f_{ii}}\frac{B_i}{B_j}.$$

  Substituting this equality into $\alpha_i\alpha_j$ that satisfies (A.2),

$$\alpha_i\alpha_j = (\tau_{ij}\tau_{ji})^{\sigma}(\theta_{ij}\theta_{ji})^{\sigma-1}\left(\frac{V_i(\varphi_{ii}^*)V_j(\varphi_{jj}^*)}{V_i(\varphi_{ij}^*)V_j(\varphi_{ji}^*)}\right) > 1.$$

  The inequality follows from $\varphi_{ij}^* > \varphi_{ii}^*$ and noting that $V_i(\varphi^*)$ is strictly decreasing in φ^*.

- The second property is that $\alpha_i = R_{ii}/R_{ij}$. Using (1), $R_{ij} = M_i^e\int_{\varphi_{ij}^*}^{\varphi_{\max}} r_{ij}(\varphi)dG_i(\varphi)$ is given by

$$R_{ij} = M_i^e\sigma w_i f_{ij}(\varphi_{ij}^*)^{1-\sigma}V_i(\varphi_{ij}^*). \tag{A.3}$$

  The result follows from substituting (A.3) into the equality of (A.2).

- The third property is that $\lambda_{ji}, \tilde{\lambda}_{ji}$ and μ_i are written in terms of α_i. By definition,

$$\begin{aligned} \lambda_{ji} &= \frac{\tau_{ji}R_{ji}}{R_{ii}+\tau_{ji}R_{ji}} = \frac{\tau_{ji}R_{ij}}{R_{ii}+\tau_{ji}R_{ij}} = \frac{\tau_{ji}}{\alpha_i+\tau_{ji}},\\ \tilde{\lambda}_{ji} &= \frac{\lambda_{ji}}{\tau_{ji}(1-\lambda_{ji})+\lambda_{ji}} = \frac{1}{\alpha_i+1},\\ \mu_i &= \frac{\tau_{ji}}{\tau_{ji}(1-\lambda_{ji})+\lambda_{ji}} = \frac{\alpha_i+\tau_{ji}}{\alpha_i+1}. \end{aligned} \tag{A.4}$$

  This follows from the second property and the TB condition.

A.5 Proof of β_i

We show the derivation of (8). Taking the log and totally differentiating W_i in (4),

$$\hat{W}_i = \frac{1}{\rho}\hat{\mu}_i + \hat{\varphi}_{ii}^* + \frac{\hat{L}_i}{\sigma-1}.$$

To express $\hat{\mu}_i$ in terms of $\hat{\varphi}_{ii}^*$, taking the log and totally differentiating μ_i in (A.4),

$$\begin{aligned} \hat{\mu}_i &= -\left(\frac{(\tau_{ji}-1)\alpha_i}{(\alpha_i+\tau_{ji})(\alpha_i+1)}\right)\hat{\alpha}_i\\ &= -\left(\frac{(\tau_{ji}-1)\lambda_{ii}}{\alpha_i+1}\right)\hat{\alpha}_i, \end{aligned}$$

where the second equality comes from noting $\lambda_{ii} = \alpha_i/(\alpha_i+\tau_{ji})$ in (A.4). Furthermore, taking the log and totally differentiating α_i in (A.2),

$$\begin{aligned} \hat{\alpha}_i &= -[\sigma-1+\gamma_{ii}+(\sigma-1+\gamma_{ij})\alpha_i]\hat{\varphi}_{ii}^*\\ &= -\left(\frac{(\alpha_i+1)\beta_i}{\alpha_i}\right)\hat{\varphi}_{ii}^*, \end{aligned}$$

where the second equality comes from the definition of β_i. Expressing $\hat{\mu}_i$ in terms of $\hat{\varphi}_{ii}^*$ gives us the result.

A.6 Proof of Lemma 1

We first show the derivation of (9). From (5), (6) and (7) with $\hat{L}_i = 0$, it follows immediately that

$$\hat{B}_i + (\sigma-1)\hat{\varphi}^*_{ii} = \sigma\hat{w}_i, \tag{A.5}$$
$$\hat{B}_j + (\sigma-1)\hat{\varphi}^*_{jj} = \sigma\hat{w}_j, \tag{A.6}$$
$$\hat{B}_j + (\sigma-1)\hat{\varphi}^*_{ij} = \sigma\hat{w}_i, \tag{A.7}$$
$$\hat{B}_i + (\sigma-1)\hat{\varphi}^*_{ji} = \sigma\hat{w}_j + (\sigma-1)\hat{\theta}_{ji}, \tag{A.8}$$
$$\hat{\varphi}^*_{ij} = -\alpha_i\hat{\varphi}^*_{ii}, \tag{A.9}$$
$$\hat{\varphi}^*_{ji} = -\alpha_j\hat{\varphi}^*_{jj}, \tag{A.10}$$
$$\hat{w}_i - \hat{w}_j = -\beta_i\hat{\varphi}^*_{ii} + \beta_j\hat{\varphi}^*_{jj}. \tag{A.11}$$

Note that (A.5)-(A.11) are the system of seven equations with seven unknowns where we have chosen $w_j = 1$ and hence $\hat{w}_j = 0$. From (A.5), (A.8), (A.10), (A.11) and (A.6), (A.7), (A.9), (A.11) respectively,

$$(\rho+\beta_i)\hat{\varphi}^*_{ii} - (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj} = -\rho\hat{\theta}_{ji},$$
$$-(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + (\beta_j+\rho)\hat{\varphi}^*_{jj} = 0,$$

where $\beta_i - \rho\alpha_i = \frac{\alpha_i}{\alpha_i+1}[\sigma - 1 - \rho + \gamma_{ii} + (\sigma - 1 - \rho + \gamma_{ij})\alpha_i] > 0$. Solving for $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ and subsequently substituting them into (A.11) yields (9). Then,

$$\frac{d\varphi^*_{ii}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{jj}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ij}}{d\theta_{ji}} > 0,\ \frac{d\varphi^*_{ji}}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} > 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} > 0.$$

Further, from (8), we have that $dP_i/d\theta_{ji} > 0$ and $dP_j/d\theta_{ji} > 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{d\theta_{ji}} > 0,\ \frac{d\varphi^*_{jj}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ij}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ji}}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} < 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} = 0,$$

and, from (8), we have that $dP_i/d\theta_{ji} < 0$ and $dP_j/d\theta_{ji} > 0$. The derivation similarly applies to τ_{ij}.

Next, we show the impact of fixed trade costs. Differentiating (1), (2) and (3) with respect to f_{ji},[19]

$$\hat{\varphi}^*_{ii} = -\frac{(\beta_j+\rho)\xi_{ji}}{\sigma\Xi}\hat{f}_{ji},$$
$$\hat{\varphi}^*_{jj} = -\frac{(\beta_i-\rho\alpha_i)\xi_{ji}}{\sigma\Xi}\hat{f}_{ji},$$
$$\hat{w}_i = \frac{\rho(\beta_i+\alpha_i\beta_j)\xi_{ji}}{\sigma\Xi}\hat{f}_{ji},$$

where $\xi_{ji} \equiv \frac{1-G_j(\varphi^*_{ji})}{J_j(\varphi^*_{ji})+1-G_j(\varphi^*_{ji})} > 0$. Similarly, differentiating the equilibrium conditions with respect to f_{ij},

$$\hat{\varphi}^*_{ii} = -\frac{(\beta_j-\rho\alpha_j)\xi_{ij}}{\sigma\Xi}\hat{f}_{ij},$$
$$\hat{\varphi}^*_{jj} = -\frac{(\beta_i+\rho)\xi_{ij}}{\sigma\Xi}\hat{f}_{ij},$$
$$\hat{w}_i = -\frac{\rho(\beta_j+\alpha_j\beta_i)\xi_{ij}}{\sigma\Xi}\hat{f}_{ij}.$$

[19]Note that f_{ji} enters the FE condition (2) when $i = j$. I would like to thank Takanori Shimizu for pointing this out.

Finally, we show the impact of import tariffs. Following similar steps, the impact of τ_{ji} is given by

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\beta_j + \rho}{\Xi}\hat{\tau}_{ji}, \\ \hat{\varphi}^*_{jj} &= -\frac{\beta_i - \rho\alpha_i}{\Xi}\hat{\tau}_{ji}, \\ \hat{w}_i &= \frac{\rho(\beta_i + \alpha_i\beta_j)}{\Xi}\hat{\tau}_{ji}, \end{aligned} \tag{A.12}$$

while the impact of τ_{ij} is given by

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\beta_j - \rho\alpha_j}{\Xi}\hat{\tau}_{ij}, \\ \hat{\varphi}^*_{jj} &= -\frac{\beta_i + \rho}{\Xi}\hat{\tau}_{ij}, \\ \hat{w}_i &= -\frac{\rho(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\tau}_{ij}. \end{aligned}$$

The above expressions show that reduction in any trade costs on exports and imports raises φ^*_{ii} and φ^*_{jj}, but starting from a symmetric situation (i.e., $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$), the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country. The only difference between them is that reduction in *import* costs $\theta_{ji}, f_{ji}, \tau_{ji}$ *reduces* w_i, whereas reduction in *export* costs $\theta_{ij}, f_{ij}, \tau_{ij}$ *raises* w_i.

A.7 Proof of Lemma 2

We first show the derivation of (11). While (A.9) and (A.10) are the same, (5) and (7) with $\hat{\theta}_{ji} = 0$ imply

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ii} = \sigma\hat{w}_i, \tag{A.13}$$
$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{jj} = \sigma\hat{w}_j, \tag{A.14}$$
$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{ij} = \sigma\hat{w}_i, \tag{A.15}$$
$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma\hat{w}_j, \tag{A.16}$$
$$\hat{w}_i - \hat{w}_j = -\beta_i\hat{\varphi}^*_{ii} + \beta_j\hat{\varphi}^*_{jj} - \hat{L}_i. \tag{A.17}$$

Note that (A.9), (A.10), (A.13)-(A.17) are the system of seven equations with seven unknowns where $\hat{w}_j = 0$. From (A.10), (A.13), (A.16), (A.17) and (A.9), (A.14), (A.15), (A.17) respectively,

$$\begin{aligned} (\beta_i + \rho)\hat{\varphi}^*_{ii} - (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj} &= -\hat{L}_i, \\ -(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + (\beta_j + \rho)\hat{\varphi}^*_{jj} &= \hat{L}_i. \end{aligned}$$

Solving for $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ and subsequently substituting them into (A.17) yields (11).

Next, we show the derivation of (13). Substituting $\hat{L}_i = -(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}$ above into (8),

$$\begin{aligned} \hat{W}_i &= \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}^*_{ii} + \frac{1}{\sigma - 1}(-(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}) \\ &= \frac{1}{\rho}\left((1 - \lambda_{ii})\beta_i - \lambda_{ii}\frac{\beta_i}{\alpha_i} + \rho - \frac{\beta_i + \rho}{\sigma}\right)\hat{\varphi}^*_{ii} + \frac{1}{\sigma - 1}(\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj} \\ &= \frac{1}{\sigma - 1}\left((\sigma - 1)(\beta_i + \rho) - \sigma\beta_i\left(\frac{\alpha_i + 1}{\alpha_i + \tau_{ji}}\right) - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\right)\hat{\varphi}^*_{ii}, \end{aligned}$$

where the second equality comes from rewriting $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ in (A.4) and the third equality comes from rewriting the first two relationships in (11) as

$$\hat{\varphi}^*_{jj} = -\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\hat{\varphi}^*_{ii}.$$

Finally, we show that starting from a symmetric situation and free trade, market expansion unambiguously improves welfare for country i. Evaluating (13) at $\alpha_i = \alpha_j, \beta_i = \beta_j$ and $\mu_i = 1$,

$$\hat{W}_i = -\frac{1}{\sigma - 1}\left(\beta_i - (\sigma - 1)\rho + (\beta_i - \rho\alpha_i)\right)\hat{\varphi}^*_{ii},$$

where $\beta_i - (\sigma - 1)\rho > 0$. The desired result follows from $\hat{\varphi}^*_{ii} < 0$. Together with (5) and (6),

$$\frac{d\varphi^*_{ii}}{dL_i} < 0,\ \frac{d\varphi^*_{jj}}{dL_i} > 0,\ \frac{d\varphi^*_{ij}}{dL_i} > 0,\ \frac{d\varphi^*_{ji}}{dL_i} < 0,\ \frac{dB_i}{dL_i} > 0,\ \frac{dB_j}{dL_i} < 0,\ \frac{dw_i}{dL_i} > 0.$$

Further, from (8), we have that $dP_i/dL_i < 0$ and $dP_j/dL_i < 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{dL_i} = 0,\ \frac{d\varphi^*_{jj}}{dL_i} = 0,\ \frac{d\varphi^*_{ij}}{dL_i} = 0,\ \frac{d\varphi^*_{ji}}{dL_i} = 0,\ \frac{dB_i}{dL_i} = 0,\ \frac{dB_j}{dL_i} = 0,\ \frac{dw_i}{dL_i} = 0,$$

and, from (8), we have that $dP_i/dL_i < 0$ and $dP_j/dL_i = 0$.

A.8 Proof of Proposition 1

We first show the derivation of (14). Taking the log and differentiating W_i in (4) with respect to τ_{ji},

$$\begin{aligned}\hat{W}_i &= \frac{1}{\rho}(\tau_{ji} - 1)\left(\frac{\alpha_i}{\alpha_i + \tau_{ji}}\right)\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii} + \frac{1}{\rho}\left(\frac{\tau_{ji}}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji} + \hat{\varphi}^*_{ii} \\ &= \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ and $\lambda_{ji} = \tau_{ji}/(\alpha_i + \tau_{ji})$ from (A.4). Compared to (8), there is an additional term that captures the terms-of-trade improvement for country i. Taking the log and differentiating (1) with respect to τ_{ji} gives the counterparts to (A.5) and (A.8). Cancelling $\hat{B}_i$ out from these and using (6) and (7) that hold for changes in τ_{ji} by setting $\hat{L}_i = 0$,

$$\hat{\tau}_{ji} = -(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}.$$

Further, noting that $\lambda_{ji} = 1 - \lambda_{ii}$ and substituting $\hat{\tau}_{ji}$ derived above,

$$\hat{W}_i = -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}(\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}. \tag{A.18}$$

Since increase in tariffs decreases φ^*_{ii} and φ^*_{jj}, (A.18) shows that tariffs in country i have a positive (negative) impact on welfare in country i by increasing (decreasing) the consumption of domestic (imported) varieties. In fact, $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ have the following relationship from (A.12):

$$\hat{\varphi}^*_{jj} = \left(\frac{\beta_i - \rho\alpha_i}{\beta_j + \rho}\right)\hat{\varphi}^*_{ii}.$$

Substituting this into (A.18) and rearranging,

$$\hat{W}_i = \frac{\beta_i - \rho\alpha_i}{\rho}\left(-\frac{\lambda_{ii}}{\alpha_i} + \frac{\lambda_{ji}(\beta_j - \rho\alpha_j)}{\beta_j + \rho}\right)\hat{\varphi}^*_{ii}.$$

Further, substituting $\lambda_{ii}/\alpha_i = \lambda_{ji}/\tau_{ji}$ from (A.4) into the above, we obtain the expression in (14).

Next, we show that starting from a symmetric situation, country i's gain from tariffs cannot compensate country j's loss. In country j that faces tariffs by country i, the effect of τ_{ji} is essentially the same as that of θ_{ji}, and changes in welfare per worker with respect to τ_{ji} are expressed as

$$\hat{W}_j = \left(\frac{(\tau_{ij} - 1)\lambda_{jj}}{\rho}\frac{\beta_j}{\alpha_j} + 1\right)\hat{\varphi}^*_{jj}.$$

Adding $\hat{W}_i$ in (A.18) and this,

$$\begin{aligned}\hat{W}_i + \hat{W}_j &= -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + \left(\frac{(\tau_{ji} - 1)\lambda_{jj}}{\rho}\frac{\beta_j}{\alpha_j} + 1 + \frac{\lambda_{ji}}{\rho}(\beta_j - \rho\alpha_j)\right)\hat{\varphi}^*_{jj} \\ &= \frac{\beta_i - \rho\alpha_i}{\rho\Xi}\left(\frac{\beta_j + \rho}{\alpha_i + \tau_{ji}} - \frac{(\tau_{ji} - 1)\beta_j}{\alpha_j + \tau_{ij}} - \rho - \frac{\tau_{ji}(\beta_j - \rho\alpha_j)}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from using (A.4) and (A.12). Notice that the first term is positive and the others are negative in the brackets, and thus changes in total welfare are in general ambiguous, as in changes in country i's welfare. However, evaluating at a symmetric situation where $\alpha_i = \alpha_j$, $\beta_i = \beta_j$ and $\tau_{ij} = \tau_{ji}$,

$$\hat{W}_i + \hat{W}_j = -\frac{\beta_i - \rho\alpha_i}{\rho\Xi}\left(\frac{(\tau_{ji} - 1)(\beta_i + \rho + \beta_i - \rho\alpha_i)}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji},$$

where the value in the brackets is positive from observing that $\tau_{ji} - 1 \geq 0$. This establishes the desired result.

Finally, we show the derivation of (15) and (16). Taking the log and differentiating W_i with respect to τ_{ji}, welfare changes can be simply expressed as

$$\hat{W}_i = \frac{\hat{\mu}_i}{\rho} + \hat{\varphi}^*_{ii},$$

which is the same as those by θ_{ji}. To show that changes can be expressed in terms of changes in λ_{ii} and μ_i, we notice that $\lambda_{ii} \times \mu_i = \alpha_i/(\alpha_i + 1)$ from (A.4). Taking the log and differentiating this with respect to τ_{ji},

$$\hat{\lambda}_{ii} + \hat{\mu}_i = -\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii}. \tag{A.19}$$

Solving for $\hat{\varphi}^*_{ii}$ and substituting it into the welfare changes gives us the expression in (15). Regarding (16), from the definition of β_i and $\tilde{\lambda}_{ji} = 1 - \tilde{\lambda}_{ii}$, β_i/α_i is given by

$$\frac{\beta_i}{\alpha_i} = \varepsilon_{ij} + (\gamma_{ii} - \gamma_{ij})(1 - \tilde{\lambda}_{ii}),$$

where $\varepsilon_{ij} \equiv \sigma - 1 + \gamma_{ij}$. Using the general expression of β_i/α_i, let us further express (15) as

$$\hat{W}_i = -\left(\frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\lambda}_{ii} + \left(\frac{1}{\rho} - \frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\mu}_i.$$

After rearranging, this can be rewritten as

$$\hat{W}_i = -\left(\frac{1}{\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij}}\right)\hat{\lambda}_{ii} - \left(\frac{\alpha_i(\gamma_{ii}-\gamma_{ij})}{(\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})((\alpha_i+1)\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})}\right)\hat{\lambda}_{ii}$$
$$+\left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij}} - \frac{\alpha_i(\gamma_{ii}-\gamma_{ij})}{(\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})((\alpha_i+1)\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})}\right)\hat{\mu}_i.$$

Further applying (A.3) to the LMC condition,

$$L_i = M_i^e \sigma \sum_{n=i,j} f_{in}(\varphi_{in}^*)^{1-\sigma} V_i(\varphi_{in}^*).$$

Taking the log and differentiating this equality with respect to τ_{ji} and using (6),

$$\hat{M}_i^e = \frac{\alpha_i}{\alpha_i+1}(\gamma_{ii}-\gamma_{ij})\hat{\varphi}_{ii}^*.$$

Solving the equality for $\hat{\varphi}_{ii}^*$ and substituting this and β_i/α_i into (A.19),

$$\hat{\lambda}_{ii} = -\left(\frac{(\alpha_i+1)\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij}}{\alpha_i(\gamma_{ii}-\gamma_{ij})}\right)\hat{M}_i^e - \hat{\mu}_i.$$

Substituting this into the second $\hat{\lambda}_{ii}$ above yields the expression $\hat{W}_i$ in (16), which becomes the same as that in Melitz and Redding (2015) without tariff revenues ($\hat{\mu}_i = 0$).

A.9 Proof of γ_{jn}

We first show the derivation of (20). Let $\phi \in \{\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji}\}$ denote a set of trade costs between two countries. From the definition of γ_{jn}, it is useful for our purpose to re-express this elasticity as a function of the productivity cutoff φ_{jn}^* for $n = i, j$:

$$\gamma_j(\varphi_{jn}^*) \equiv -\frac{d\ln V_j(\varphi_{jn}^*)}{d\ln \varphi_{jn}^*}.$$

If $\gamma_j(\varphi_{jn}^*)$ is strictly increasing (decreasing) in the productivity cutoff φ_{jn}^*, the differential is negative (positive) so long as selection into the export market is satisfied:

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \quad \Longrightarrow \quad \gamma_{jj} - \gamma_{ji} \lesseqgtr 0. \tag{A.20}$$

Thus, if the extensive margin elasticity $\gamma_{jn}^* = \gamma_j(\varphi_{jn}^*)$ is a monotonic function in the productivity cutoff φ_{jn}^*, the sign of the differential is the same for a given productivity distribution $G_j(\varphi)$. Moreover, differentiating $\varepsilon_{ji} = \sigma - 1 + \gamma_{ji}$ with respect to ϕ defined above,

$$\frac{d\varepsilon_{ji}}{d\phi} = \gamma_j'(\varphi_{ji}^*)\frac{d\varphi_{ji}^*}{d\phi}.$$

Since $\frac{d\varphi_{ji}^*}{d\phi} > 0$ from Lemma 1, so long as $\gamma_j(\varphi_{ji}^*)$ is a monotonic function of φ_{jn}^*,

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{d\phi} \gtreqless 0.$$

Next, we show that, if the differential is negative (positive), the trade elasticity is decreasing (increasing) in country i's market size, while the converse is true for country j's market size. Differentiating ε_{ji} with respect to L_i and L_j respectively and noting that $\frac{d\varphi_{ji}^*}{dL_i} < 0$ and $\frac{d\varphi_{ji}^*}{dL_j} > 0$ from Lemma 2 as well as (A.20),

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \frac{d\varepsilon_{ji}}{dL_i} \lesseqgtr 0,$$
$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \frac{d\varepsilon_{ji}}{dL_j} \gtreqless 0.$$

A.10 Proof of Proposition 2

We first show that, if the differential is negative (positive), reduction in trade costs has the impact on the optimal tariffs t_{ji}^* not only by decreasing the domestic trade share $\tilde{\lambda}_{jj}$ but also by decreasing (increasing) the trade elasticity ε_{ji}. The optimal tariffs (17) are rewritten as

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}\left(\frac{\beta_j}{\alpha_j} - \rho\right)},$$

where reduction in trade costs always decreases $\tilde{\lambda}_{jj}$ irrespective of the sign of $\gamma_{jj} - \gamma_{ji}$ from Lemma 1. Thus, it suffices to show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), β_j/α_j decreases (increases) with ϕ. For that purpose, rewrite the definition of β_j in Section 3 as

$$\frac{\beta_j}{\alpha_j} = \varepsilon_{ji} + \frac{\gamma_{jj} - \gamma_{ji}}{\alpha_j + 1}.$$

Differentiating this with respect to ϕ,

$$\begin{aligned}\frac{d(\beta_j/\alpha_j)}{d\phi} &= \gamma_j'(\varphi_{ji}^*)\frac{d\varphi_{ji}^*}{d\phi} + \frac{-\gamma_j'(\varphi_{ji}^*)\frac{d\varphi_{ji}^*}{d\phi}(\alpha_j+1) - (\gamma_{ji} - \gamma_{jj})\frac{d\alpha_j}{d\phi}}{(\alpha_j+1)^2} \\ &= \frac{\alpha_j}{\alpha_j+1}\left(\frac{d\varepsilon_{ji}}{d\phi} - \left(\frac{\gamma_{jj} - \gamma_{ji}}{\alpha_j(\alpha_j+1)}\right)\frac{d\alpha_j}{d\phi}\right).\end{aligned}$$

Using (20) and noting that $\frac{d\alpha_j}{d\phi} > 0$,[20]

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \frac{d(\beta_j/\alpha_j)}{d\phi} \gtreqless 0.$$

Next, we show that market size has a similar impact on t_{ji}^*. From the impact of market size on $\tilde{\lambda}_{jj}$ from Lemma 2, it suffices to show the impact of L_i, L_j on β_j/α_j. Differentiating β_j/α_j above with respect to L_i and L_j respectively and noting that $\frac{d\varphi_{ji}^*}{dL_i} < 0$ and $\frac{d\varphi_{ji}^*}{dL_j} > 0$ from Lemma 2,

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \frac{d(\beta_j/\alpha_j)}{dL_i} \lesseqgtr 0,$$
$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \frac{d(\beta_j/\alpha_j)}{dL_j} \gtreqless 0.$$

[20]To be precise, if $\phi = f_{ji}$, we require certain restrictions on $G_j(\varphi)$ such that $\frac{\gamma_{ji}}{\sigma-1} \geq \frac{1-G_j(\varphi_{ji}^*)}{J_j(\varphi_{ji}^*)}$. This is a sufficient condition for $\frac{d\alpha_j}{df_{ji}} > 0$ which holds under a Pareto distribution (details are available upon request to the author).

B Nash Tariffs

Suppose that each country sets a tariff rate to maximize respective welfare. We now allow country i to choose tariffs taking into account country j's retaliation against country i's tariffs and derive Nash tariffs.

Lemma 1 suggests that increase in country j's tariffs τ_{ij} always decreases the domestic trade share $\tilde{\lambda}_{jj}$. In addition, Proposition 2 shows that when $\gamma_{jj} - \gamma_{ji} < 0$, this increase also decreases the trade elasticity ε_{ji}. These jointly mean that the best response function is downward-sloping so that tariffs are strategic substitutes for one another. If $\gamma_{jj} - \gamma_{ji} > 0$ and increase in ε_{ji} is greater than decrease in $\tilde{\lambda}_{jj}$, country i's optimal tariffs are increasing in country j's tariffs. In this case, the best response functions are upward-sloping and the optimal tariffs are strategic complements for one another. As usual, the Nash tariffs τ^*_{ji}, τ^*_{ij} are determined at which the best response functions intersect in the (τ_{ji}, τ_{ij}) space, but the variable nature of the trade elasticity alters the equilibrium properties of such tariffs. Further, the Nash tariffs are bounded from above and below. If trade costs are so high that no firm exports from country i, the domestic trade share in country j approaches to unity ($\tilde{\lambda}_{jj} = 1$). Using this fact in (17), it follows immediately that the lower bound is

$$\underline{\tau}^*_{ji} = 1 + \frac{\rho\alpha_j}{\beta_j - \rho\alpha_j} = \frac{\beta_j}{\beta_j - \rho\alpha_j}.$$

Note that this bound also represents country i's optimal tariffs when country i is treated as a small economy (relative to country j). In particular, from $\beta_j/\alpha_j = k$ under a Pareto distribution, it reduces to $\underline{\tau}^*_{ji} = k/(k-\rho)$ which is the optimal tariffs in a small economy (Demidova and Rodríguez-Clare, 2009).

In contrast, if trade costs are so low that all operating firms export from country j ($\varphi^*_{jj} = \varphi^*_{ji}$), we have $\alpha_j = f_{jj}/f_{ji}$ (from the definition of α_j) and $\gamma_{jj} = \gamma_{ji}$ (from the definition of γ_{jn}) and hence $\beta_j/\alpha_j = \varepsilon_{ji}$. Using these and $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ in (17), the upper bound is

$$\overline{\tau}^*_{ji} = 1 + \frac{\rho\left(1 + \frac{f_{jj}}{f_{ji}}\right)}{\frac{f_{jj}}{f_{ji}}\left(\varepsilon_{ji} - \rho\right)} = \frac{\varepsilon_{ji} + \rho\frac{f_{ji}}{f_{jj}}}{\varepsilon_{ji} - \rho}.$$

Note that both $\underline{\tau}^*_{ji}$ and $\overline{\tau}^*_{ji}$ are variable and endogenously respond to exogenous shocks.

To better appreciate the equilibrium properties of the Nash tariffs, we follow Felbermayr et al. (2013) in assuming that the two countries are symmetric and choose their tariffs non-cooperatively. In Nash equilibrium, these countries impose the same optimal tariffs $\tau^*_{ij} = \tau^*_{ji} \equiv \tau^*$ where wages are equalized between them, i.e., $w_i = w_j \equiv w = 1$. Exploiting the symmetry, let us further define

$$\begin{aligned}
&\theta_{ij} = \theta_{ji} \equiv \theta, \quad f_{ii} = f_{jj} \equiv f_d, \quad f_{ij} = f_{ji} \equiv f_x, \quad L_i = L_j \equiv L, \quad \varphi^*_{ii} = \varphi^*_{jj} \equiv \varphi^*_d, \quad \varphi^*_{ij} = \varphi^*_{ji} \equiv \varphi^*_x, \\
&\tilde{\lambda}_{ii} = \tilde{\lambda}_{jj} \equiv \tilde{\lambda}, \quad \varepsilon_{ij} = \varepsilon_{ji} \equiv \varepsilon, \quad \gamma_{ii} = \gamma_{jj} \equiv \gamma_d, \quad \gamma_{ij} = \gamma_{ji} \equiv \gamma_x, \quad \alpha_i = \alpha_j \equiv \alpha, \quad \beta_i = \beta_j \equiv \beta.
\end{aligned}$$

Then, finding the Nash tariffs is equivalent to finding a solution to the fixed point problem $\tau = f(\tau)$ in (17) where the dependence of $f(\tau)$ on θ, f_x and L is understood:

$$f(\tau) = 1 + \frac{\rho}{\tilde{\lambda}\left(\varepsilon - (\gamma_d - \gamma_x)(1 - \tilde{\lambda}) - \rho\right)}.$$

Since all of the key endogenous variables (i.e., $\tilde{\lambda}$, ε, $\gamma_d - \gamma_x$) are a function of tariffs, the fixed point problem only implicitly characterizes the Nash tariffs as in the optimal tariffs examined in Section 4.1. Nevertheless, we can discuss several equilibrium properties of the Nash tariffs.

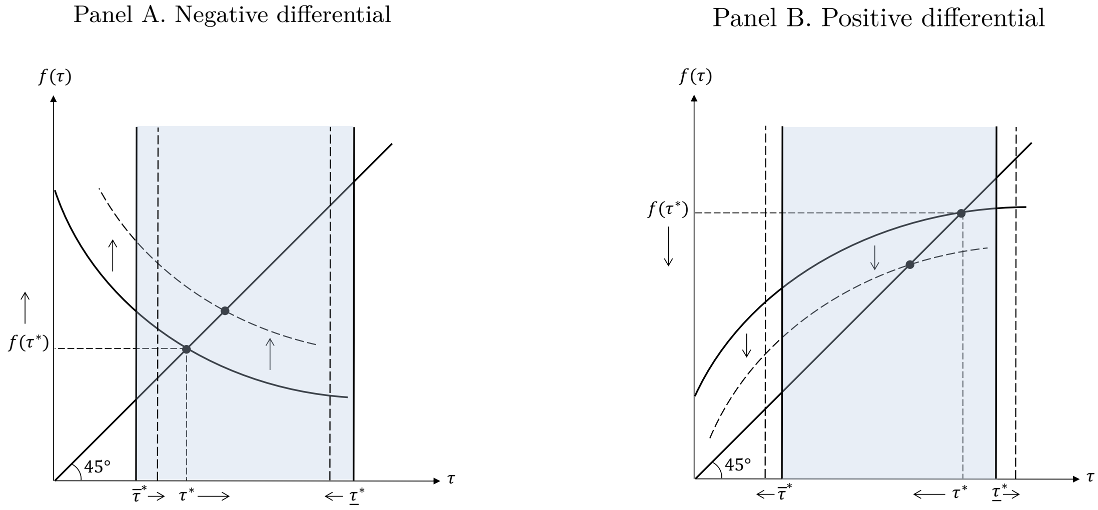

Figure 3: Effect of trade liberalization on Nash tariffs

As in optimal tariffs, the effect of exogenous shocks on Nash tariffs critically depends on the sign of $\gamma_d - \gamma_x$. When $\gamma_d - \gamma_x < 0$, both the domestic trade share $\tilde{\lambda}$ and trade elasticity ε increases with τ. In this case, $f(\tau)$ is strictly decreasing in τ, which reflects that tariffs are strategic substitutes. In contrast, when $\gamma_d - \gamma_x > 0$, $\tilde{\lambda}$ increases with τ but ε decreases with it. In this case, $f(\tau)$ is strictly increasing in τ if a rise in ε is greater than a fall in $\tilde{\lambda}$, which reflects that tariffs are strategic complements. Figure 3 depicts a 45-degree line plus a $f(\tau)$ curve for two possible cases: tariffs are strategic substitutes in Panel A and tariffs are strategic complements in Panel B. In either panel, the Nash tariffs are found at which a 45-degree line and a $f(\tau)$ curve intersect. Such tariffs lie within the shaded area in the figure where the lower and upper bounds are respectively denoted by $\underline{\tau}^*_{ij} = \underline{\tau}^*_{ji} \equiv \underline{\tau}^*$ and $\overline{\tau}^*_{ij} = \overline{\tau}^*_{ji} \equiv \overline{\tau}^*$.

Consider first the impact of trade liberalization at the symmetric situation. Reduction in trade costs (both variable θ and fixed f_x) always decreases the domestic trade share $\tilde{\lambda}$. At the same time, such reduction can affect the trade elasticity ε, depending on the sign of $\gamma_d - \gamma_x$. When the differential is negative, reduction in θ also decreases the trade elasticity ε. In this case, the $f(\tau)$ curve shifts up and the Nash tariffs τ^* are higher. Moreover, the gap between the upper and lower bounds is narrower (i.e., the Nash tariffs tend to converge) as a result of such reduction in Panel A. When the differential is positive, the converse is true in Panel B. Finally, when the differential is zero, reduction in trade costs has no impact on the trade elasticity and the Nash tariffs are higher only through a decline in the domestic trade share, whereby the two bounds are unaffected. While the impact of trade costs on the Nash tariffs are similar to that on the optimal tariffs, changes in the key equilibrium variables arise on a different scale between *bilateral* reduction in trade costs (examined here) and *unilateral* reduction in these costs (examined in Section 4.2). In case of variable trade costs θ, for example, solving the system of three equations ((5), (6)) at the symmetric situation for three unknowns $(\hat{\varphi}^*_d, \hat{\varphi}^*_x, \hat{B})$,

$$\hat{\varphi}^*_d = -\frac{1}{\alpha+1}\hat{\theta}, \quad \hat{\varphi}^*_x = \frac{\alpha}{\alpha+1}\hat{\theta}.$$

Comparing this and (9) reveals that reduction in variable trade costs has different impacts on the cutoffs, even if both changes are evaluated at the symmetric situation, reflecting the fact that both countries reduce variable trade costs here while only country i reduces such costs in (9).

Consider next the impact of market size at the symmetric situation. Bilateral expansion in market size (L) has no impact on the domestic trade share, because market size has no effect on the productivity cutoffs with the equalized wage, i.e., $\hat{w}_i = \hat{w}_j = 0$ (see (12)). Noting that the extensive margin elasticities are a function of these cutoffs, this also means that market size has no effect on the trade elasticity and the extensive margin elasticity differential. Consequently, the $f(\tau)$ curve does not shift at all, so that the Nash tariffs and the two bounds also remain unchanged. In contrast to trade liberalization above, this impact of market size necessarily holds irrespective of the sign of the differential.

One of the key upshots of our argument is that the optimal trade policy can be substantially mis-estimated even in an environment in which countries choose tariffs non-cooperatively. This is of particular importance for assessment of the optimal trade policy in globalization where reductions in transportation or communication costs are significant. The model shows that, whenever the trade elasticity differs across markets, there is an additional channel through which trade costs affect the optimal trade policy, i.e., endogenous trade elasticity. In fact, recent work using firm-level data has identified the empirical relevance of this aspect. For example, estimating trade flows in their generalized gravity equation, Helpman et al. (2008) find substantial variation in the trade elasticity with respect to observable trade costs (proxied by distance) between country-pairs, which indicates that the trade elasticity is not unique in reality. Calibrating their heterogeneous firm model into US firm-level data, Melitz and Redding (2015) also show that missing the variable nature of the trade elasticity can give rise to a quantitatively large discrepancy between the predicted and "true" welfare gains from trade liberalization. In the context of trade policy, these insights into welfare gains imply that the Nash tariffs can be mis-estimated if the governments fail to take account of the micro structure that makes the trade elasticity variable in their policy making.

Proposition 3 *Evaluating at a symmetric situation, the Nash tariffs have the following equilibrium properties:*

(i) *When the extensive margin elasticity is the same between domestic and export markets, reduction in trade costs increases the Nash tariffs only through decreases in the domestic trade share.*

(ii) *When the extensive margin is more (less) elastic in an export market than in a domestic market, they reinforce (attenuate) the impact on the Nash tariffs through decreases (increases) in the trade elasticity.*

(iii) *Regardless of the sign of the extensive margin elasticity differential, market size has no impact on the Nash tariffs.*

We close the section by mentioning the quantitative relevance of Proposition 3. Using the parameter values in Section 5 and applying our analytical solutions to the Nash tariffs, we are able to quantify the Nash tariffs, albeit under the limited situation where the two symmetric countries bilaterally change exogenous variables. Since the indirect effect through changes in wages disappears in that case, market size has no impact on the Nash tariffs, regardless of the trade elasticity is constant or not. Even if the indirect effect is absent, however, trade costs have an impact on productivity cutoffs, as seen above. Furthermore, the impact is always greater for a variable trade elasticity than for a constant elasticity (where the former occurs whenever $\gamma_d - \gamma_x \neq 0$), because the trade elasticity endogenously responds to changes in trade costs. Exploiting the model's outcome, we can examine the impact of trade costs on the Nash tariffs to reach the following conclusion: the impact is *quantitatively* stronger with a variable trade elasticity than with a constant trade elasticity, though the impact is *qualitatively* similar for one another.

C Numerical Solutions

Our calibration procedures closely follow Melitz and Redding (2015). We first consider the heterogeneous firm models with variable and constant trade elasticities and compare the optimal tariffs in (17) and (18). Then we consider the heterogeneous and homogeneous firm models with a constant trade elasticity and compare the optimal tariffs in (18) and (19). Following Felbermayr et al. (2013), the two countries are assumed to differ in their tariff rate but are otherwise identical in an initial equilibrium where all exogenous variables are the same in both cases. For simplicity, we use the short-hand notations introduced in Appendix B (e.g., $\theta_{ij} = \theta_{ji} \equiv \theta$).

Comparison between (17) and (18) We choose the elasticity of substitution between varieties $\sigma = 4$ and hence $\rho = 0.75$. We set the shape parameter of a Pareto distribution $k = 4.25$, the scale parameter $\varphi_{\min} = 1$, and the upper bound either $\varphi_{\max} = 2.85$ in (17) or $\varphi_{\max} = \infty$ in (18).

We follow Melitz and Redding (2015) in calibrating trade costs to match the average fraction of exports in firm sales in US manufacturing (which is 0.14 as reported by Bernard et al. (2007)). In contrast to their study that matches this number to variable trade costs only, we also consider tariffs and hence $\frac{\tau^{-\sigma}\theta^{1-\sigma}}{1+\tau^{-\sigma}\theta^{1-\sigma}} = 0.14$. We set τ equal to 1.045 which matches the world applied tariff rate (weighted mean, all products in 2002), where the world tariff rate is obtained from the World Bank Data for the same year as Bernard et al. (2007). Together with $\sigma = 4$, this implies $\theta = 1.7$. Regarding fixed costs, we set $f_d = f_e = 1$ while $f_x = 0.535$ for bounded Pareto and $f_x = 0.545$ for unbounded Pareto; see Melitz and Redding (2015) for detailed discussions. Regarding market size, we set $L = 170$ to make the effects of θ and L easily comparable.

Using these parameter values and specifications of the distribution, we can uniquely determine the values of equilibrium variables. In our numerical exercise, we do this by solving the two equations. One system of equations is the share of firms that export in each country, which is given as $\chi \equiv [1 - G(\varphi_x^*)]/[1 - G(\varphi_d^*)]$. Under the Pareto distribution, this share is expressed in terms of φ_d^*, φ_x^* along with distributional parameters:

$$\chi = \frac{\left(\frac{\varphi_{\min}}{\varphi_x^*}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}{\left(\frac{\varphi_{\min}}{\varphi_d^*}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

Another system of equations for the unknowns φ_d^*, φ_x^* is selection into exporting. Evaluating the ZCP condition in (1) at the symmetric situation, selection into exporting in (A.1) implies

$$\left(\frac{\varphi_x^*}{\varphi_d^*}\right)^{\sigma-1} = \frac{\tau^\sigma \theta^{\sigma-1} f_x}{f_d}.$$

Solving these two relationships for the two unknowns, φ_d^*, φ_x^*, the former is expressed as

$$(\varphi_d^*)^{-k} = \frac{\varphi_{\max}^k (1-\chi)}{\tau^{-\frac{k\sigma}{\sigma-1}} \theta^{-k} \left(\frac{f_x}{f_d}\right)^{-\frac{k}{\sigma-1}} - \chi}.$$

The average share of firms that export in US manufacturing is 0.18 (Bernard et al., 2007) and hence $\chi = 0.18$. Further, plugging the calibrated parameter values yields the values of two unknowns in the initial equilibrium under the bounded Pareto distribution: $\varphi_d^* = 1.16$, $\varphi_x^* = 1.70$. Note that the values of the two cutoffs are not uniquely determined under the unbounded Pareto distribution where $\varphi_{\max} = \infty$. Once they are determined, the values of other key endogenous variables are automatically pinned down, as shown in the main text.

We compare (17) and (18) holding φ_d^*, φ_x^* determined above equal across the different models, where $w = 1$ in the initial equilibrium. The key endogenous variables in these optimal tariff formulas are

$$\varepsilon_x = \sigma - 1 + \gamma_x,$$

$$\gamma_n = (k - (\sigma - 1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_n^*}\right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_n^*}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}},$$

$$\tilde{\lambda} = \frac{\alpha}{\alpha + 1},$$

where $n = d, x$. Even conditioning on φ_d^*, φ_x^*, the values of three moments are different between unbounded and bounded Pareto distributions where $\varphi_{\max} = \infty$ and $\varphi_{\max} < \infty$, respectively. The trade elasticity ε_x is k under an unbounded Pareto distribution, whereas it is greater than k under a bounded Pareto distribution. Similarly, the extensive margin elasticity differential $\gamma_d - \gamma_x$ is zero when $\varphi_{\max} = \infty$, while it is negative when $\varphi_{\max} < \infty$ so long as $\varphi_x^* > \varphi_d^*$. Finally, the domestic trade share $\tilde{\lambda}$ is different because (A.1) and (A.2) imply

$$\alpha = \tau^\sigma \theta^{\sigma-1} \frac{V(\varphi_d^*)}{V(\varphi_x^*)}.$$

Applying the Pareto distribution to $V(\varphi^*)$ in Appendix A.4, we get

$$V(\varphi^*) = \frac{k\varphi_{\min}^k}{k - (\sigma - 1)} \frac{\varphi^{*-(k-(\sigma-1))} - \varphi_{\max}^{-(k-(\sigma-1))}}{1 - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k},$$

which takes different values, depending on whether $\varphi_{\max} = \infty$ or $\varphi_{\max} < \infty$. Using the values of productivity cutoffs, we can quantify the three key moments of optimal tariffs in the initial equilibrium. These differences lead to different values of the optimal tariffs in the initial equilibrium, given as the dots in Figure 2.

Furthermore, the analytical solutions of comparative statics outcomes in Section 3 allow us to address the quantitative impact of unilateral changes in trade costs and market size on the optimal tariffs. As we examine the effect of unilateral changes in the exogenous variables, we must depart from the symmetric situation in the initial equilibrium for the comparative statics. For this reason, the country subscripts i, j are re-attached to relevant variables below and examine the effect of unilateral changes in exogenous variables from the initial equilibrium. Consider the effect of θ_{ji} where country i unilaterally changes variable trade costs of importing from country j. Evaluating (9) at the symmetric situation $\alpha_i = \alpha_j = \alpha$, $\beta_i = \beta_j = \beta$ and using (6), changes in the productivity cutoffs in country j from the initial equilibrium are

$$\hat{\varphi}_{jj}^* = -\frac{\rho(\beta - \rho\alpha)}{\Xi} \hat{\theta}_{ji},$$

$$\hat{\varphi}_{ji}^* = \frac{\rho\alpha(\beta - \rho\alpha)}{\Xi} \hat{\theta}_{ji}.$$

Thus, starting from the symmetric situation, 1 percent reduction in θ_{ji} leads to $\frac{\rho(\beta-\rho\alpha)}{\Xi}$ percent increase in φ_{jj}^* and $\frac{\rho\alpha(\beta-\rho\alpha)}{\Xi}$ percent decrease in φ_{ji}^* respectively. Using the calibrated values in the initial equilibrium, we can compute changes in φ_{jj}^* and φ_{ji}^* from the initial equilibrium. These changes are then used to compute changes in the three key moments of optimal tariffs for changes in the optimal tariffs. Note that, depending on whether $\varphi_{\max} = \infty$ or $\varphi_{\max} < \infty$, not only is α but also β and Ξ take different values, and so do $\hat{\varphi}_{jj}^*, \hat{\varphi}_{ji}^*$. This generates different changes in optimal tariffs in (17) or (18), given as the curves in Figure 2.

Comparison between (18) and (19) We keep the parameters in the heterogeneous firm model the same as for the unbounded Pareto distribution and so does (18). As for (19), we choose the degenerate distribution in the homogeneous firm model so that the two models generate the same aggregate variables in the initial equilibrium. Let $\tilde{\varphi}_d^*$ and $\tilde{\varphi}_x^*$ denote (exogenous) productivity domestic and export cutoffs in the homogenous firm model. To meaningfully compare the two different models, we choose the values of these cutoffs so that $\varphi_d^* = \tilde{\varphi}_d^*$ and $\varphi_x^* = \tilde{\varphi}_x^*$ in the initial equilibrium. Under the condition, the aggregate equilibrium outcomes, including the share of firms that export χ and the domestic trade share $\tilde{\lambda}$, are the same in the initial equilibrium (Melitz and Redding, 2015). Despite that, the value of the optimal tariffs is different between these models in the initial equilibrium. This is simply because the trade elasticity ε_x comes from the the intensive margin elasticity $\sigma - 1$ and the extensive margin elasticity γ in (18), while it comes only from the intensive margin elasticity in (19). As the domestic trade share $\tilde{\lambda}$ is the same between these models, this difference implies that as long as $k > \sigma - 1$ (which ensures that average firm size is finite under the unbounded Pareto distribution), the optimal tariffs are lower for (18) than for (19) in the initial equilibrium, given as the dots in Figure 2.

Further, changes in the optimal tariffs are different, since the productivity cutoffs endogenously respond to changes in exogenous variables in the heterogeneous firm model, while they are constant in the homogenous firm model. This difference implies that the heterogeneous firm model has an additional adjustment margin that is absent in the homogenous firm model, which critically affects welfare gains (Melitz and Redding, 2015). In our policy context, the difference implies that the optimal tariffs in the heterogeneous firm model respond to changes in exogenous variables more sharply than those in the homogenous firm model through changes in the productivity cutoffs. This explains why changes in the optimal tariffs are greater for (18) than for (19) from the initial equilibrium, given as the curves in Figure 2.