Trade with Firm-to-Firm Relationships: Identifying New Gains and Losses from Trade

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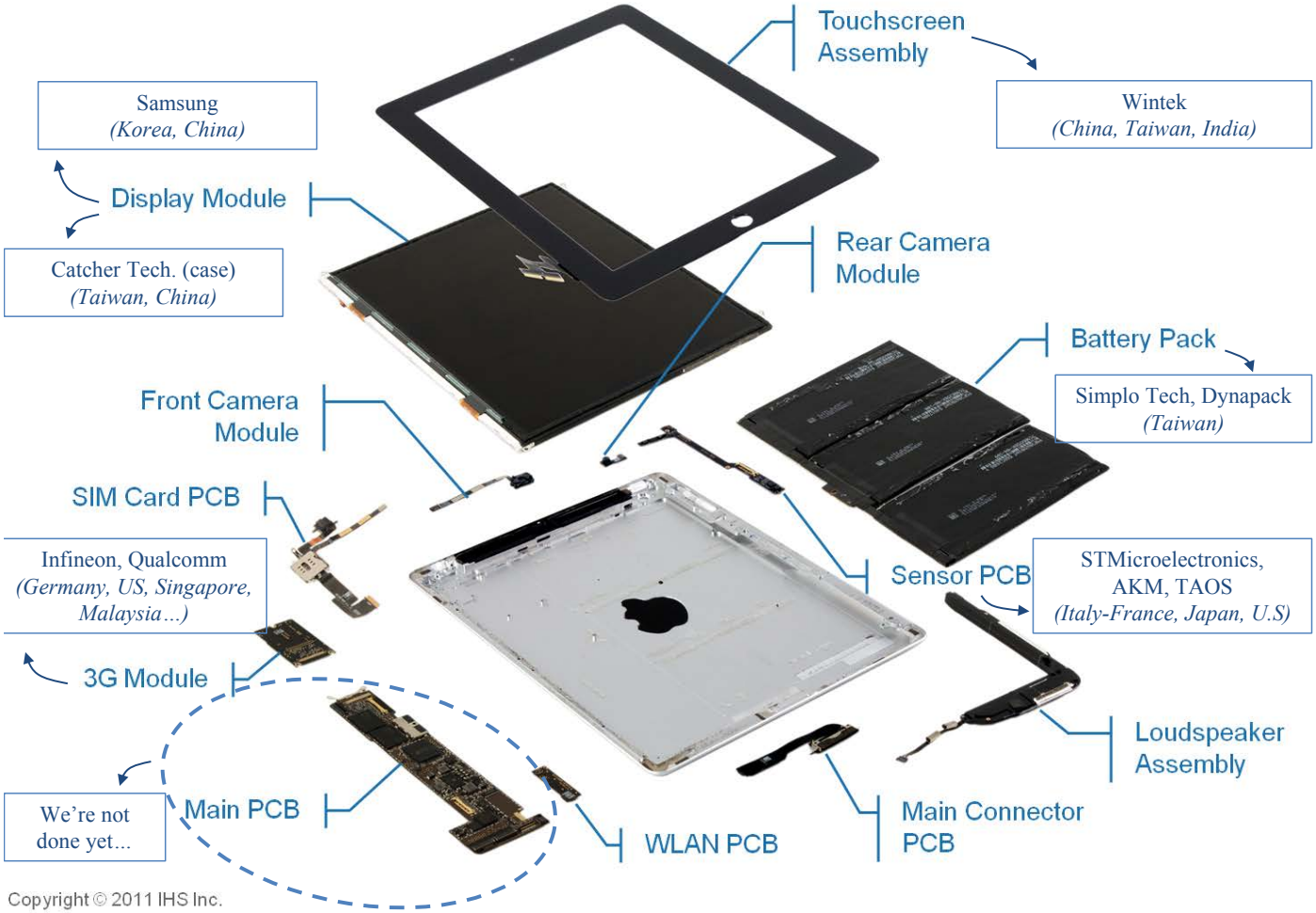
Motivation

- Firms often search for suppliers to procure specialized inputs:
  1. While a few core inputs are made in-house, other non-core inputs are largely purchased from outside suppliers
  2. IT revolution makes it easier to search for suppliers not only within borders but also across borders
  3. Access to a wide range of outsourced inputs improves production technology of firms

  $\Rightarrow$ Consider Apple's sourcing strategy

Motivation



$\Rightarrow$ Firm-to-firm relationships may seriously affect welfare

Key results

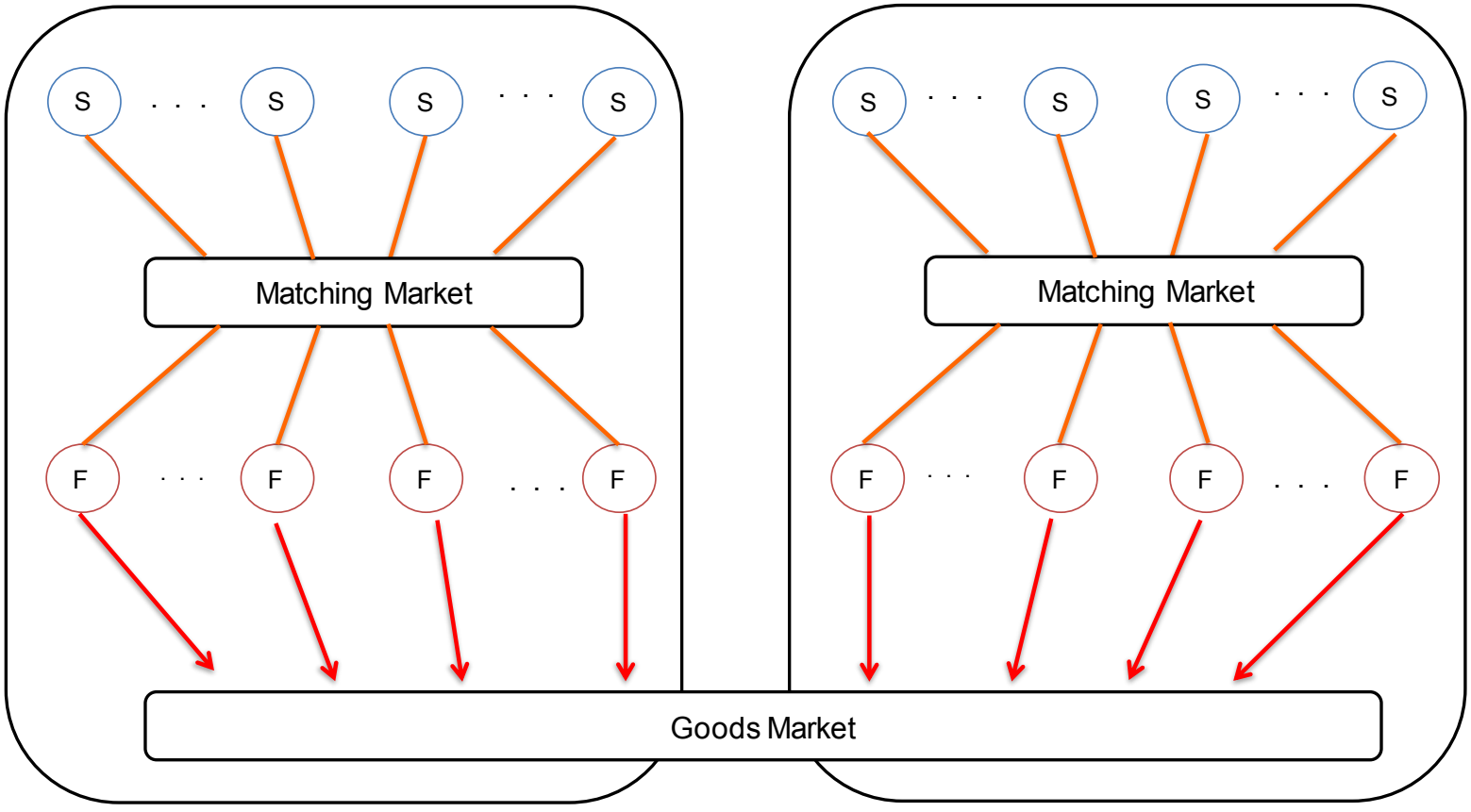
- Question
  - How and why does the formation of firm-to-firm relationships affect the welfare gains and losses from trade?

- Answers
  1. Welfare changes are computed as

$$d \ln W = -\frac{1}{\varepsilon}\left(d \ln s - d \ln u^F \right)$$

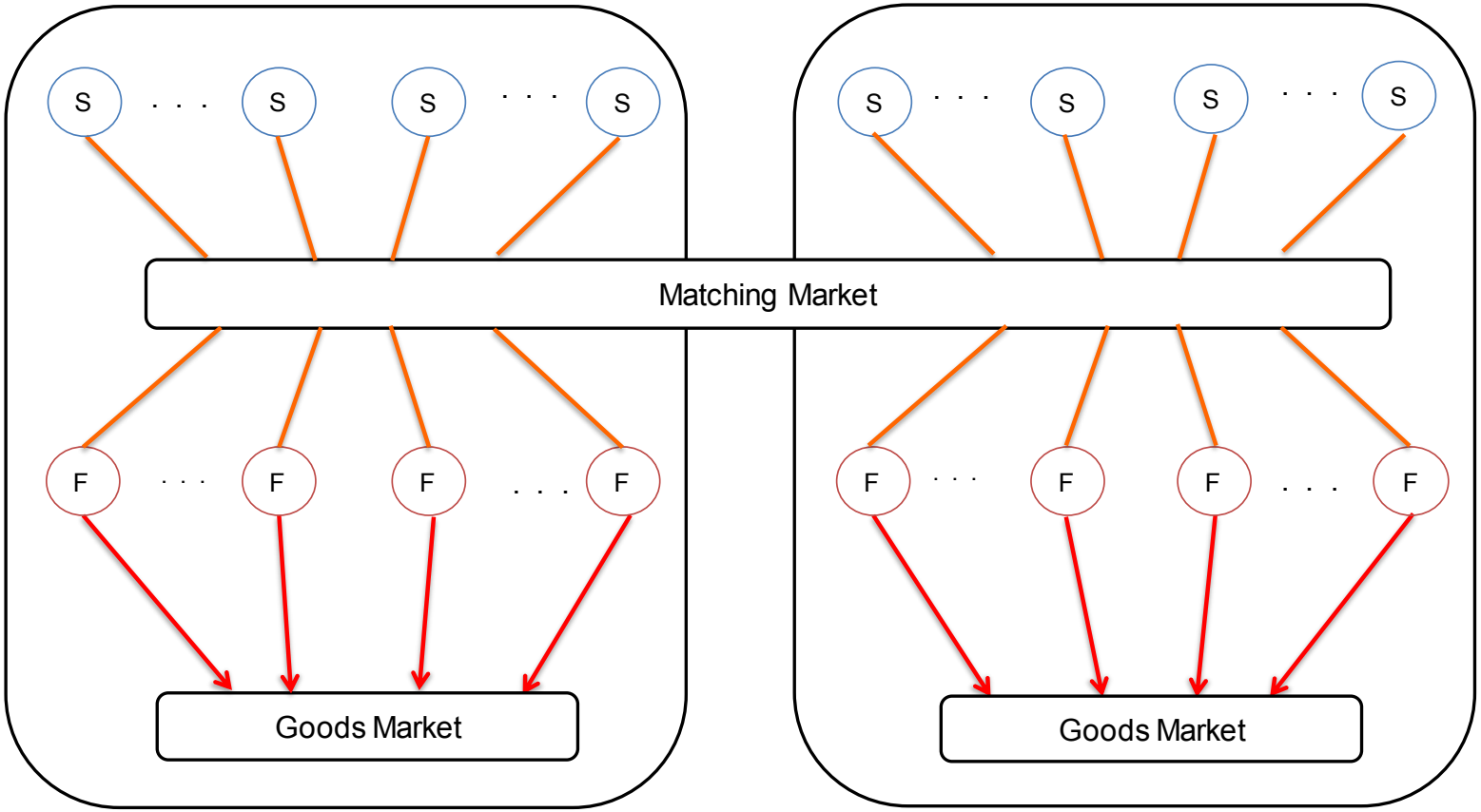
     - s is the expenditure share on goods produced by unmatched firms
     - u^F is the mass of unmatched firms
  2. Welfare gains and losses depend on whether trade integrates goods markets or matching markets across countries

Key results



Goods market integration $\Rightarrow$ Welfare gains are amplified

Key results



Matching market integration $\Rightarrow$ Welfare losses may occur

Related literature

- Key assumptions:
  1. Firms and suppliers randomly match and bargain over generated surplus (Felbermayr et al., 2011; Arkolakis et al., 2025)
  2. Firms and suppliers have one-to-one relationships in their search process (Krolikowski and McCallum, 2021; Sugita et al., 2021)
  3. Matched firms can enjoy a love-of-variety effect from an input expansion (Ethier, 1982; Halpern et al., 2015)

Outline of this talk

- Setup
  - Homogeneous firm model (Krugman, 1980) with search and matching
  - Firm-to-firm relationships increase productivity
- Goods market integration
  - *X-integration* hereafter
  - New welfare channels
- Calibration
  - Quantify the effects of firm-to-firm relationships
  - Quantitative departures from ACR

Setup

- Consumer preferences:

$$U = \left(\int_{\omega} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$

- Demand and expenditure for variety ω:

$$y(\omega) = Ap(\omega)^{-\sigma}$$
$$r(\omega) = Ap(\omega)^{1-\sigma}$$

where A is the index of industry demand

Setup

- Firm technology:

$$y(\omega) = \left((x^F(\omega))^{\frac{\sigma-1}{\sigma}} + \mathbb{1}(\omega)(x^S(\omega))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

  where both inputs are produced competitively

- Firm unit cost:

$$c(\omega) = \left((wa^F)^{1-\sigma} + \mathbb{1}(\omega)(wa^S)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \frac{wa^F}{\varphi(\omega)}$$

  where

$$\varphi(\omega) \equiv \left(1 + \mathbb{1}(\omega) \left(\frac{a^F}{a^S} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}$$

Setup

- Profit-maximization problem:

$$\max_{x^F(\omega),\, x^S(\omega)} \; r(\omega) - wa^F x^F(\omega) - \mathbb{1}(\omega) wa^S x^S(\omega) - \mathbb{1}(\omega) w(f^F + f^S)$$

- Optimal pricing, revenue and profit:

$$p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{wa^F}{\varphi}$$

$$r(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \frac{\varphi}{wa^F} \right)^{\sigma - 1}$$

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - \mathbb{1}(\varphi) wf$$

where $f \equiv f^F + f^S$

Setup

- Matching function:

$$m(u^F, u^S)$$

  which satisfies CRS in matching

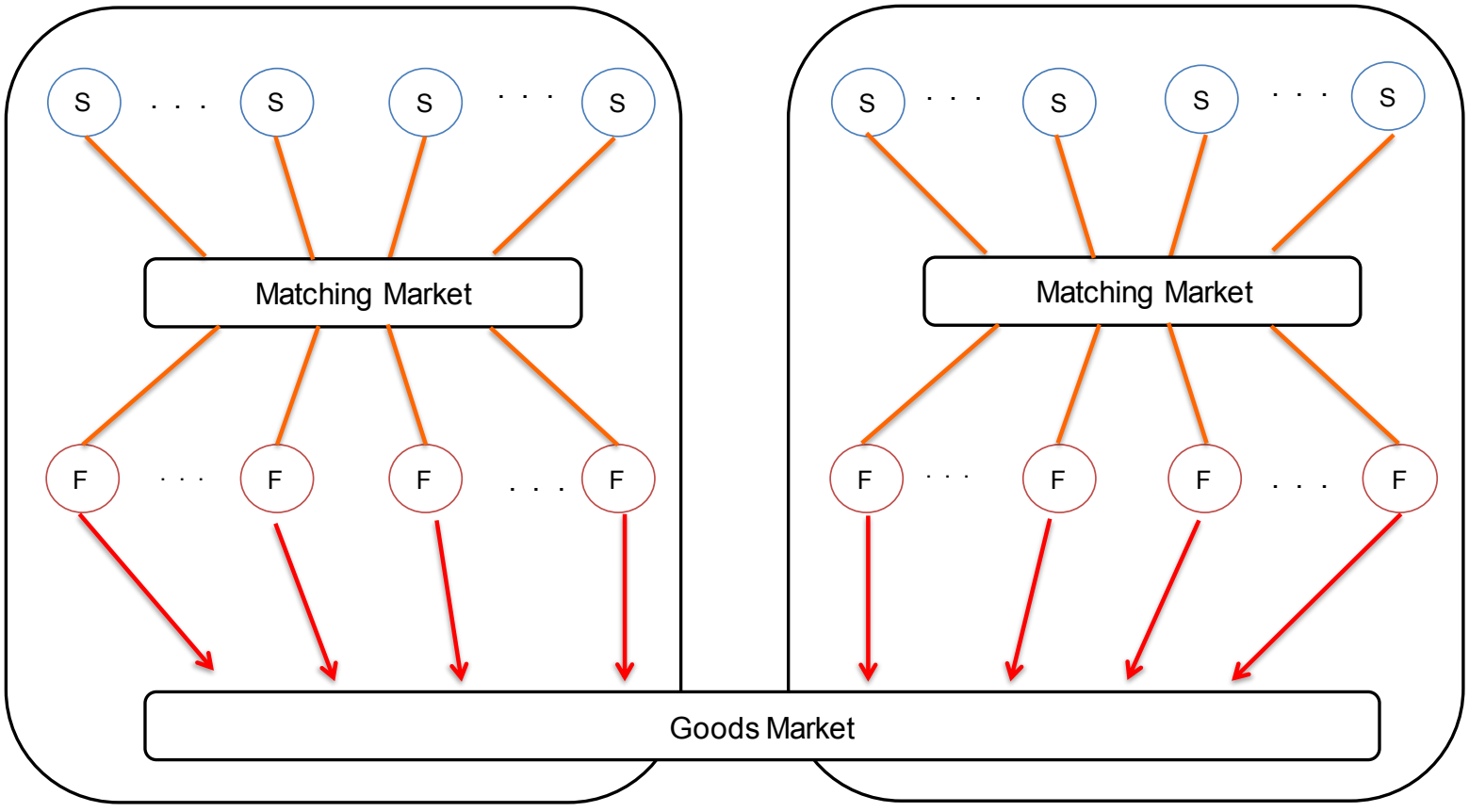
- Probability of matches:

$$\mu^F \equiv m(u^F, u^S)/u^F = m(1, \theta)$$
$$\mu^S \equiv m(u^F, u^S)/u^S = m(1/\theta, 1) = \mu^F/\theta$$

  where $\theta \equiv u^S/u^F$

- One-to-one matching: $\underbrace{N^F - u^F}_{\text{Mass of matched firms}} = \underbrace{N^S - u^S}_{\text{Mass of matched suppliers}} \equiv n$

X-integration



Depending on trade costs, unmatched firms can or cannot export

X-integration

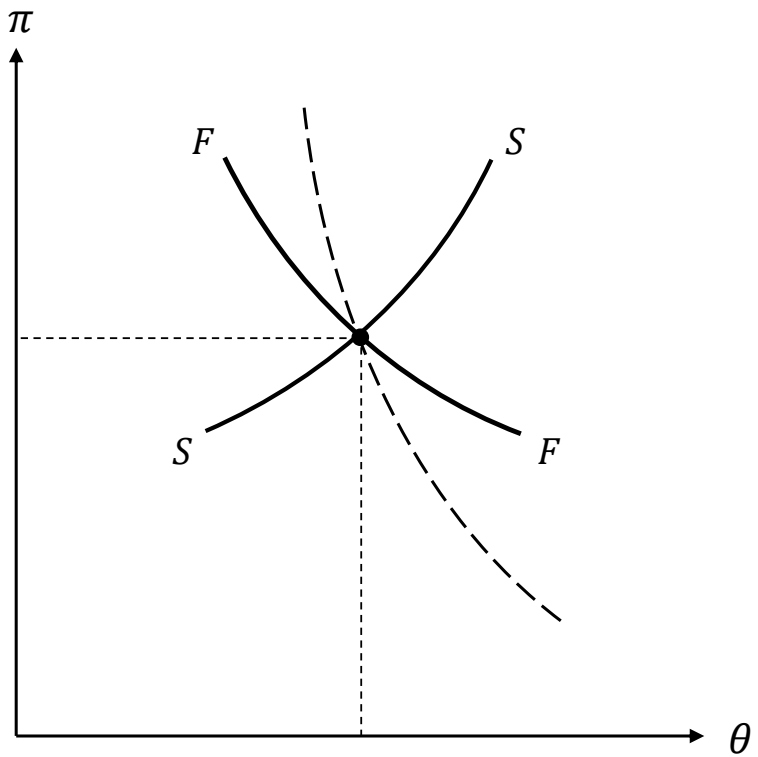
- When both matched and unmatched firms export, the free entry condition of firms and suppliers are, respectively, written as

$$\pi - f_x + \frac{n}{N^F}\beta\left(\pi(\varphi) - \pi\right) = f_e^F$$
$$\frac{n}{N^S}(1-\beta)\left(\pi(\varphi) - \pi\right) = f_e^S$$

  where

  - π: unmatched profit
  - $\pi(\varphi)$: matched profit
  - $\pi(\varphi) - \pi$: surplus generated by matches

X-integration



$$\theta = u^S/u^F = (N^S - n)/(N^F - n)$$

- *FF* curve

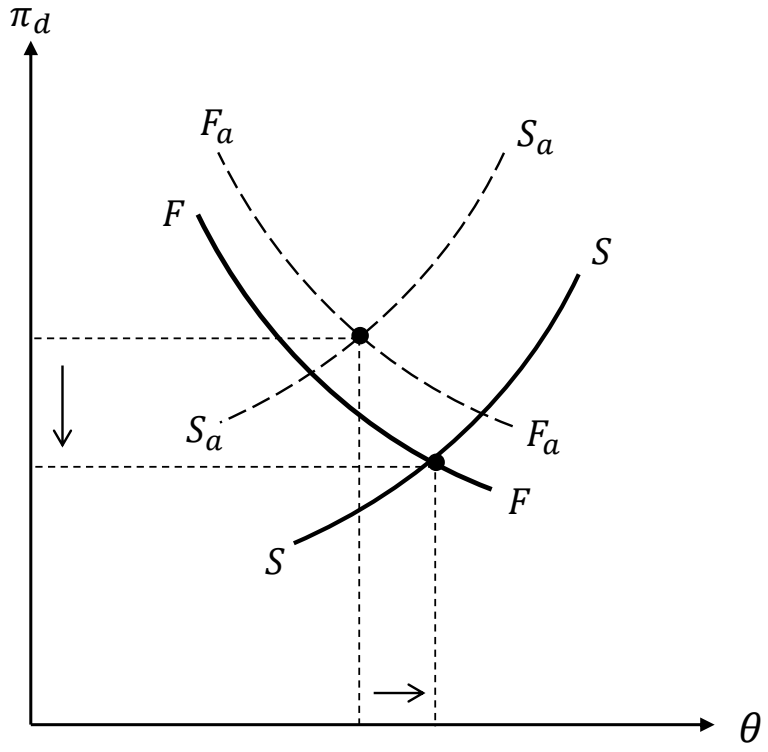
  $\theta \uparrow \Rightarrow \ \mu^F \uparrow \Rightarrow \ \pi \downarrow$

- *SS* curve

  $\theta \uparrow \Rightarrow \ \mu^S \downarrow \Rightarrow \ \pi \uparrow$

- θ and π are consistent with free entry in X-integration equilibrium

X-integration



- Matched firms get a larger surplus in X-integration, inducing new entry
- Impact of X-integration

$$\pi < \pi_a$$

$$\theta > \theta_a$$

X-integration

- Gains from trade (GFT) in X-integration:
  1. $\pi < \pi_a \Longrightarrow$ Resources are reallocated from (inefficient) unmatched firms to (efficient) matched firms
  2. $\theta > \theta_a \Longrightarrow$ Firms have the higher probability to meet suppliers, enhancing overall production efficiency of the industry

X-integration

- Sufficient statistics for welfare:
  - Trade elasticity

$$\varepsilon \equiv -\frac{\partial \ln(R_x/R_d)}{\partial \ln \tau_x} = \sigma - 1$$

  - Domestic expenditure share

$$\lambda \equiv \frac{R_d}{R} = \frac{1}{1+\tau_x^{1-\sigma}}$$

  - Domestic unmatched expenditure share

$$s \equiv \frac{(N^F - n)r_d}{R} = \frac{1}{(1+\tau_x^{1-\sigma})\left(1+\varphi^{\sigma-1}\mu^F/\delta\right)}$$

X-integration

- Welfare changes in X-integration are computed as

$$d \ln W = -\frac{1}{\varepsilon}\left(d \ln s - d \ln u^F \right)$$

  - Without firm-to-firm relationships, $s = \lambda$ and $u^F = N^F (= L/\sigma f_e^F)$, the changes are $d \ln W = -d \ln \lambda / \varepsilon$ (Arkolakis et al., 2012)
  - With firm-to-firm relationships, $d \ln s < d \ln \lambda$ and $d \ln u^F \lesseqgtr 0$, and the changes are amplified by such relationships
  - **Takeaway**: In search and matching, we need to use variables of *unmatched* firms

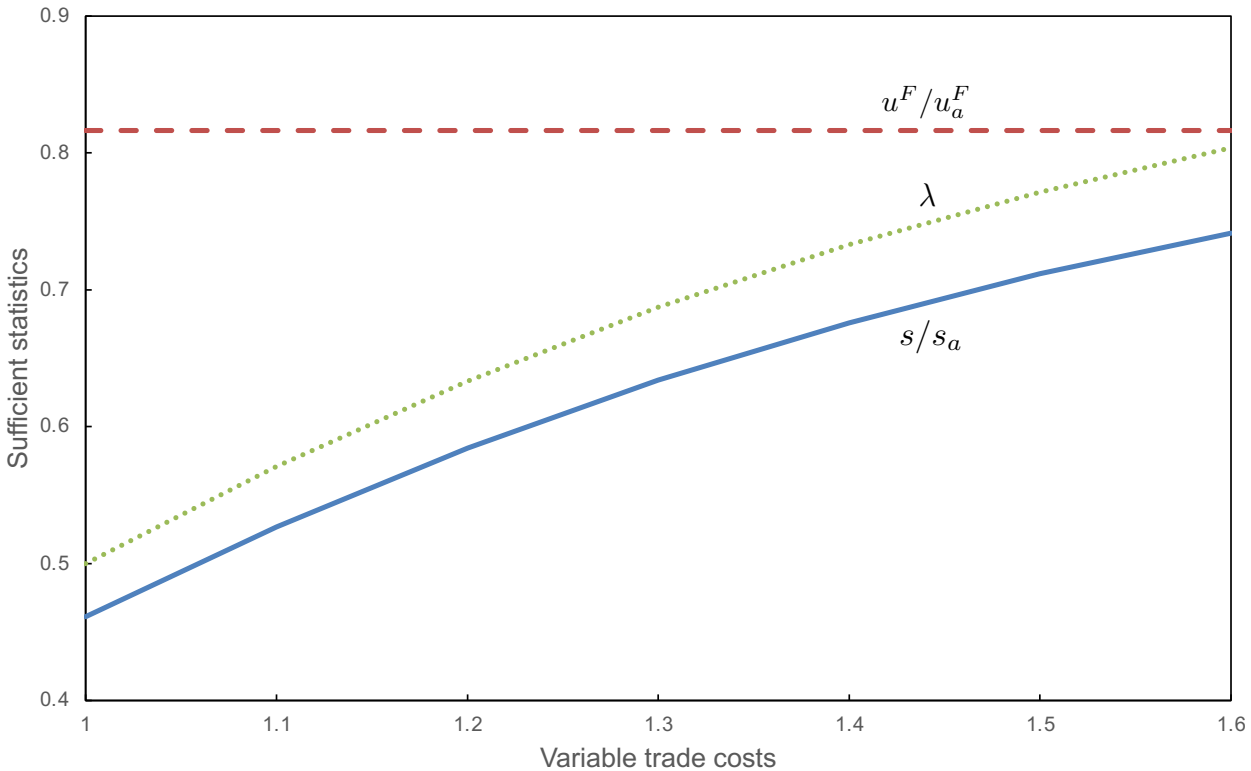
Calibration

- Firms and suppliers meet through the following matching function

$$m(u^F, u^S) = \left(\left(u^F\right)^{-\iota} + \left(u^S\right)^{-\iota} \right)^{-1/\iota}$$

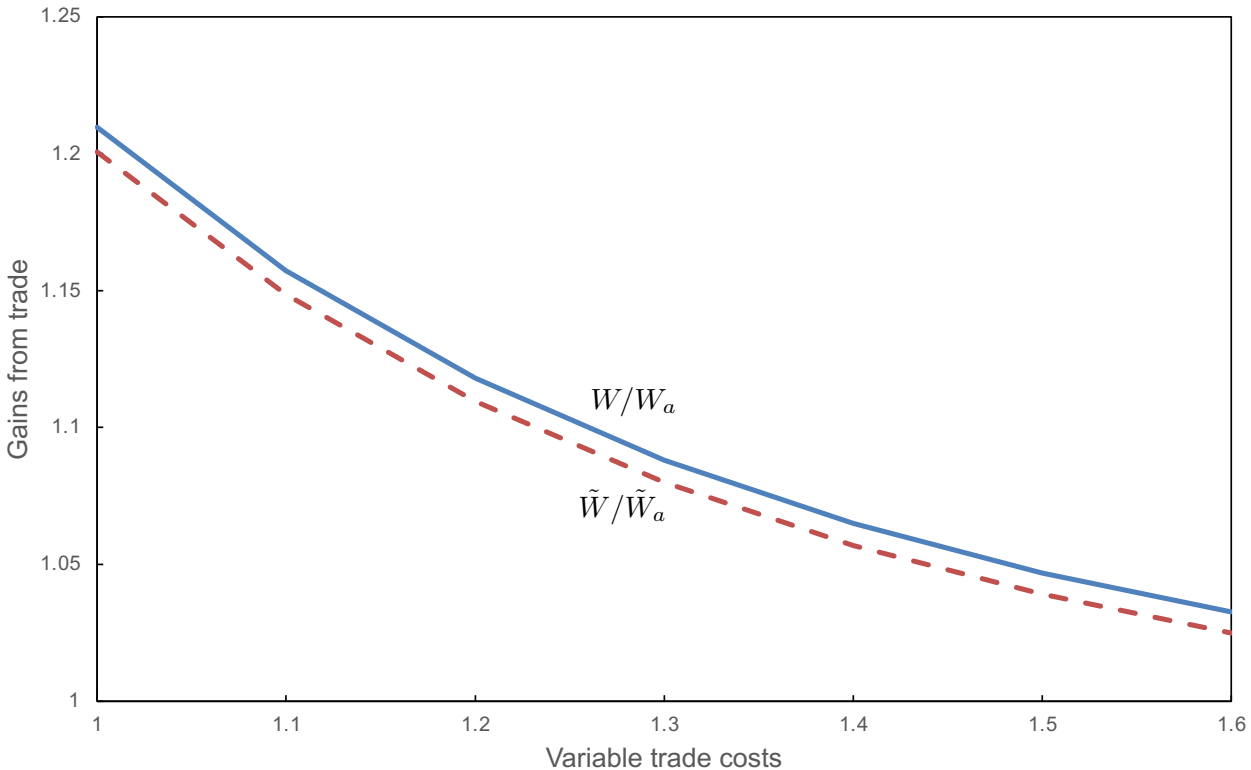
  - Market tightness u^F/u^S is 0.66 and matching elasticity ι is 0.45 $\Rightarrow$ $\mu^F = 0.26$, $\mu^S = 0.17$ (Heise, 2024)
  - Imported inputs increase firm productivity by about 25 percent $\Rightarrow$ $\varphi = 1.25$ (Halpern et al., 2015)
  - Standard values based on central estimates of US data, e.g., $\sigma = 4$, $\tau_x = 1.6$

Calibration



Sufficient statistics for welfare

Calibration



Welfare gains from trade

Summary

- Key messages:
  - With firm-to-firm relationships, welfare changes can be computed by sufficient statistics of *unmatched* agents
  - Whether such relationships affect welfare gains or losses depends on wether trade integrates goods markets or matching markets