Trade Elasticities and Welfare Losses from Tariffs under Vertical Oligopoly*

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Abstract

How do tariffs affect trade elasticities and welfare losses when vertically related sectors are oligopolistic? To address the question, this paper develops a vertical oligopoly model in which one country specializing in final production sets tariffs on an intermediate good from another country specializing in input production. Using constant-elasticity demand, we find that increased tariffs simultaneously force firms and suppliers to exit in each country, raising trade elasticities in oligopolistic sectors relative to those in competitive sectors. The difference also gives rise to a different effect of tariffs on welfare, causing severe welfare losses under vertical oligopoly relative to those under perfect competition. Using the Chinese customs data, we provide evidence on the relationship between market concentration and trade elasticities predicted by our model.

Keywords: Trade elasticities, welfare losses from tariffs, co-movement, vertical oligopoly
JEL Classification Numbers: F12, F13, F14

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1 Introduction

Recent years have witnessed faster growth of intermediate goods in world trade volume. It is often said that rapid input trade has been triggered by *vertical specialization* where each country engages only in a particular stage of production sequences spread across the globe. Such specialization on a global scale is also occasionally referred to as *global value chains* (GVCs) where each country completely specializes in production at each stage.[1] In analyzing firms' decision about how and where to purchase intermediate goods from suppliers, most work in the relevant literature has focused on the role of either search and matching between firms and suppliers, or relationship specificity and contract incompleteness among firm-to-firm relationships in input price negotiation.[2] In practice, however, a large portion of intermediate goods is also traded in the markets where the input price is determined at the market-clearing level, which equates the total demand of firms to the total supply of input produced by suppliers.

Our approach to vertical specialization and GVCs with market-based transactions has several unique features that cannot be dealt with by existing trade models in this literature. First, our paper is able to shed light on the role of large firms that dominate trade flows in international fragmentation of production. There are lots of studies documenting the increasing strategic interdependences across firms which are unlikely to be of measure zero relative to the markets. Head and Spencer (2017) find that the markets have been becoming concentrated towards top 4 firms over time in several industries, suggesting that market structure is characterized by oligopoly. However, little attention has been paid to how such firms play a role in vertical specialization and GVCs in the literature by assuming perfectly or monopolistically competitive firms at each stage of production. Second, our paper is able to analyze the impact of tariffs on different margins of input trade in vertically related industries. Again, many studies show that the interdependencies across countries are crucial for understanding trade policy. Antràs et al. (2025) find that, as multinational firms source input from affiliated suppliers in different countries, trade policy shocks can lead to similar effects on countries involved in GVCs through a complementary force on the extensive margin. However, most work in this literature focuses on the response of foreign direct investment to trade policy changes, and it has been less known about policy implications of foreign outsourcing where firms purchase input from independent suppliers in the markets through which trade policy changes are propagated in adjustment of the extensive and intensive margins of input trade.

This paper develops a vertical oligopoly model where firms and suppliers from different stages of production operate through the markets. Following the literature on vertical specialization, we assume that one country (say, Home) produces a final good in a downstream sector, whereas another country (say, Foreign) produces an intermediate good in an upstream sector. Moreover, following evidence that homogeneous intermediate goods tend to be traded in the markets (Nunn, 2007), we assume that two countries produce homogeneous goods and the input price is determined at the market-clearing level. A Home government sets tariffs on Foreign input to influence the terms-of-trade, recognizing that Home final production rests on such input. In this model setting, we examine the long-run effect of tariffs on total input imports, thereby distinguishing the extensive margin and the intensive margin of input trade. In so doing, we investigate the production side peculiar to oligopoly (i.e., strategic interactions among firms) as well as the demand side (i.e., elasticity and convexity of demand) in a unified framework, which proves useful for understanding how recent tariff hikes affect international trade in which a small number of large firms play a key role via input-output linkages across nations.

[1] See Yi (2003, 2010) for the importance of vertical specialization. Antràs and de Gortari (2020) and Johnson and Moxnes (2023) examine this kind of specialization in a framework of GVCs.

[2] For papers in this literature, see, in alphabetical order, Acemoglu et al. (2007); Antràs (2003); Antràs and Helpman (2004); Costinot (2009); Grossman and Helpman (2002); Grossman et al. (2024); Ornelas and Turner (2008, 2012).

We show that increased tariffs by the Home government force Home firms as well as Foreign suppliers to exit the downstream and upstream sectors, respectively. On the other hand, such tariffs force them to produce at small scale in the respective sectors only if the consumer demand is strictly convex. Thus, the *extensive* margin is decreasing in tariffs, whereas the *intensive* margin depends on the demand condition. Intuition behind the contrasting effect of tariffs is explained as follows. Consider first the extensive margin. Increased tariffs raise the production costs of all producers participating in vertical specialization, reducing the profits of Home firms and Foreign suppliers. When the market structure is endogenous via free entry, the decreased probability forces these producers to exit the respective sectors. It is important to emphasize that the effect of tariffs is unique to our *vertical* oligopoly model where Home's final good and Foreign's intermediate good are *complements*: tariffs lead to *co-movement*, thereby forcing Home firms and Foreign suppliers to exit in each country simultaneously. Our result stands in sharp contrast to *horizontal* oligopoly models where traded goods are *substitutes*: tariffs lead to *de-location*, thereby shifting entry of firms from a liberalizing country to a non-liberalizing country (e.g., Bagwell and Staiger, 2012). Hence, our model shows that the long-run effect of tariffs on entry and exit might not hold in vertical specialization.

In contrast, increased tariffs can increase or decrease the intensive margin, as a decrease in total input imports (by such tariffs) can be more or less than proportionate to a decrease in the extensive margin. For example, consider Home firms' intensive margin. Increased tariffs on Foreign input decrease Home firms' input demand. With input-output linkages, increased tariffs negatively affect not only Foreign suppliers' input production but also Home firms' output production by raising their production costs. In addition to this, increased tariffs force Home firms and Foreign suppliers to exit the respective sectors. As Foreign suppliers exit more, the amount of input produced by them is smaller and thus the input price determined at the market-clearing level is higher, which prompts Home firms to decrease their output. However, as Home firms exit more, surviving Home firms face less intense competition, which increases Home firms' output. Due to these opposing forces, the effect of tariffs on Home firms' intensive margin is generally ambiguous. However, under constant-elasticity demand that exhibits strict convexity of demand, we find that Home firms' intensive margin always decreases with tariffs. The effect of tariffs applies to Foreign suppliers' intensive margin, in the sense that this intensive margin always decreases with tariffs under constant-elasticity demand.[3]

When increased tariffs decrease both the extensive and intensive margins in the vertically related sectors, how does this affect the trade elasticity, i.e., the elasticity of imports with respect to trade costs? As shown by Arkolakis et al. (2012), this elasticity is one of the sufficient statistics for welfare changes associated with any trade shocks. Thus, it is crucial to address how vertical oligopoly affects this sufficient statistic in computing welfare losses by increased tariffs. Using constant-elasticity demand once again, we find that the trade elasticity with respect to tariffs under vertical oligopoly is strictly larger than that under the competitive benchmark. Intuition closely relates to co-movement of producers by tariffs between the upstream and downstream sectors: increased tariffs force Home firms and Foreign suppliers to exit the respective sectors via input-output linkages. This additional margin, which is absent in the competitive model, contributes to the trade elasticity increase. We also find that the difference gives rise to a different effect of tariffs on welfare, causing severe welfare losses under vertical oligopoly relative to those under perfect competition by the same reason: tariff-induced exit of Home firms (resp. Foreign suppliers) elevates the production costs of the final (resp. intermediate) good, which, in turn, increases the prices of both goods and therefore hurts consumers even further.

[3] These results on the extensive and intensive margins bear resemblance to those in the recent literature of heterogeneous firms. One of the differences is that a response of the intensive margin to variable trade costs (tariffs here) depends on the demand side (i.e., demand curvature) in the present model; while it depends on the supply side (i.e., firm distribution) in most existing models. We will elaborate on this point in the literature review below.

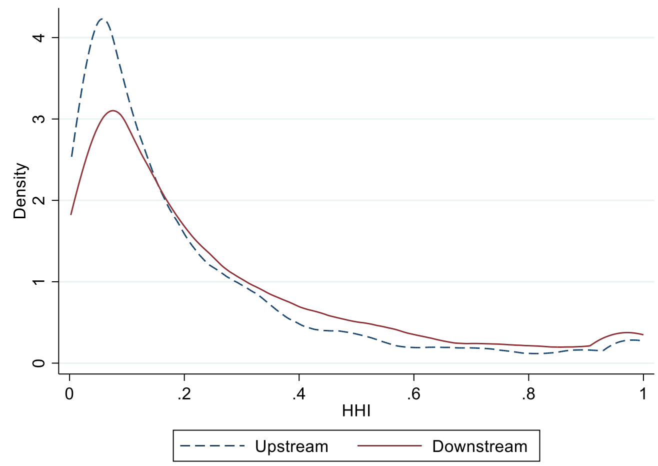

Figure 1: Product-level market concentration and upstreamness

Source: Authors' own calculations based on transaction-level data from Chinese Customs and the Input-Output (IO) industry-level upstreamness index constructed by Chor et al. (2021).

Notes: The HHI is calculated at the Harmonized System (HS) 6-digit product level, while the upstreamness index is defined at the IO 3-digit industry level. Using a concordance table, we map HS 6-digit products to IO 3-digit industries. Higher values of the HHI indicate greater market concentration. Products are classified as upstream or downstream based on whether their upstreamness is above or below the median value (2.34) of the index.

Our vertical oligopoly model with constant-elasticity demand has a key prediction that is empirically testable: the market is more oligopolistic, the larger the trade elasticity. To assess the empirical relevance of this finding, we focus on input trade by classifying traded goods into final and intermediate goods, and measure the degree of oligopoly by either the Herfindahl-Hirschman Index (HHI) or the four-firm concentration ratio (CR4) at the 6-digit HS product level in the Chinese transaction-level import data. We find that the effects of input tariffs on total input imports (in terms of import value and quantity) depend on market concentration. Specifically, a one-percentage-point reduction in input tariffs increases the import value of high-HHI products by 1.6%. The effect is even larger for import quantity: the same tariff reduction leads to a 6.2% increase in the import quantity of high-HHI products. Moreover, the heterogeneous effects are even more pronounced when using CR4 as the measure of concentration. These results suggest that imported intermediate goods in more concentrated markets are more sensitive to tariff changes.

The key premise of our paper is that the vertically related industries are oligopolistic. As an illustration of the potential empirical relevance of our model framework, using detailed import data from Chinese Customs, Figure 1 displays the kernel density distribution of the HHI of imports for two groups of products: upstream and downstream. The figure shows that both upstream and downstream products exhibit right-skewed distributions – most products have low to moderate concentration (HHI < 0.3), while only a few are highly concentrated. However, the differences between the two distributions are relatively minor, whereby the curves have similar shapes and tails. At the upper end (HHI > 0.8), both distributions show small density bumps, indicating that some products in both groups are highly concentrated. While this finding primarily applies to the product level, similar evidence on the vertically related pattern is also observed at a more aggregated industry level: highly concentrated industries can be found at both the upstream and downstream stages of the production process (see Table A.1 in Appendix).

Our paper contributes to the literature of vertical specialization and GVCs. Most papers in this literature consider perfectly competitive firms in a multistage production process where production stages are completed sequentially. For example, Antràs and de Gortari (2020) study the specialization pattern of participant countries within GVCs that is endogenously determined with trade barriers, while Johnson and Moxnes (2023) find that input trade is more sensitive to changes in trade costs than final goods trade due to the endogenous reorganization of GVCs.[4] Our model setting is simpler than that of previous work in that we consider two production sectors (i.e., upstream and downstream sectors) but try to shed new light on the aspect ignored by the existing studies: imperfect competition and strategic interactions among producers in vertically related sectors. As the number of producers is endogenously determined by free entry in these sectors, our framework allows us to analyze the effect of tariffs on the extensive and intensive margins, which is key to understanding why market concentration matters for the trade elasticity in vertical specialization.

A handful of papers have analyzed vertical relationships in oligopoly. Ishikawa and Spencer (1999) examine the effect of export subsidies for an imported intermediate good on social welfare in vertically related markets with a fixed number of firms. Ghosh and Morita (2007) explore social efficiency in terms of the number of firms in a vertical oligopoly model with free entry. These papers, however, do not consider the effect of policy changes on the input trade margins. On the other hand, using single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985) and, more recently, Etro (2011), Bagwell and Staiger (2012) all show that the effect of tariffs may be substantially altered when invoking the free entry condition in oligopolistic markets. We extend this valuable insight to a vertical oligopoly setting and examine the long-run effect of tariffs on entry and exit in the context of vertical specialization and GVCs.

Demand-side conditions, like the elasticity and convexity of demand, play an important role in determining the impact of tariffs on input trade margins in the vertical oligopoly framework. This finding is reminiscent of Mrázová and Neary (2017) who highlight how the demand structure critically shapes the degree of pass-through under monopoly and monopolistic competition. Our vertical oligopoly model is able to offer a richer context with strategic interactions across producers and input-output linkages across countries. However, unlike theirs, we do not consider general demand functions, but primarily focus on a constant-elasticity demand function, which enables us to provide precise characterizations of the equilibrium of vertical oligopoly under free entry. We also show that demand-side conditions are critical for the effect of tariffs on the two trade margins (and on the trade elasticity) in our oligopoly model, which differs from most of monopolistic competition models where supply-side conditions are important for this effect.

Our empirical test of the trade elasticity is intimately related to one of the main empirical tests conducted by Antràs and de Gortari (2020) and Antràs et al. (2024): the trade elasticity is larger for downstream than for upstream sectors in GVCs. Given that downstream sectors tend to be less competitive than upstream sectors (Antràs and Chor, 2022), our finding has similar flavors to theirs: the trade elasticity is larger in sectors with more concentration. Despite that aspect, however, there are critical differences: the trade-elasticity results are obtained under conditions of *perfect* competition in their papers and *oligopolistic* competition in our paper. This difference leads to potentially different mechanisms between the models. In Antràs and de Gortari (2020), imports are more sensitive to trade costs for downstream than for upstream sectors, since the costs accumulate ("compound") towards final production along GVCs. In this paper, the same result occurs because trade costs gives rise to co-movement of producers in vertically-related oligopolistic sectors, which yields larger effects on imports in this model than those in the competitive model without such co-movement.

[4]See Antràs and Chor (2022) for a comprehensive literature review on GVCs. To the best of our knowledge, vertical oligopoly has not been analyzed in the GVC literature, including their survey paper.

2 Model

This section develops a model of vertical oligopoly with two countries, say Home and Foreign, that specialize in producing final and intermediate goods, respectively. The goal of this paper is to characterize the equilibrium of vertical specialization and to explore the impact of trade costs (notably tariffs) on trade elasticities and welfare in the presence of strategic interactions across producers and input-output linkages across nations.

Section 2 is organized as follows. We start by describing the model setup of vertical oligopoly in Section 2.1. We solve for equilibrium with a fixed number of producers to examine the effect of trade costs in Section 2.2. Then, we introduce free entry in the model to disentangle the effect of trade costs on the intensive and extensive margins in Section 2.3. Using these equilibrium characterizations, we finally analyze the welfare effect of tariffs in the model with a fixed or endogenous number of producers via free entry by mentioning policy implications from our analysis in Section 2.4.

2.1 Setup

Consider first the structure of consumer preferences and demand in the two countries. Home has a mass L of identical consumers who share the same quasi-linear utility function, $U \equiv U(Q)+y$, where Q is an aggregate final good produced under conditions of oligopoly while y is a freely-tradable numeraire produced under conditions of perfect competition. As is well-known, y equalizes wage rates to 1 between the countries. Further, to focus on the effect of import tariffs on Home welfare, Foreign is assumed to consume only y. Utility maximization of Home consumers generates an inverse demand function for the final good Q, which is specified as

$$P \equiv P(Q) = AQ^{-1/\sigma}, \quad \sigma > 1. \tag{1}$$

Using constant-elasticity demand in (1), the elasticity and curvature of demand are, respectively, given by

$$\begin{aligned} \varepsilon &\equiv -\frac{P(Q)}{QP'(Q)} = \sigma, \\ \rho &\equiv \frac{QP''(Q)}{P'(Q)} = -\left(\frac{\sigma+1}{\sigma}\right). \end{aligned} \tag{2}$$

(2) implies that ρ approaches its minimum (resp. maximum) when $\sigma \to 1$ (resp. $\sigma \to \infty$). Thus, $-2 < \rho < -1$. Though an admissible combination of (ε, ρ) is narrower under this specific demand than under general demands, (1) is quite common in the trade literature,[5] and turns out remarkably useful for equilibrium characterizations of vertically related sectors.

On the production side, the two countries engage in vertical specialization: Home (Foreign) specializes in producing the final (intermediate) good in the downstream (upstream) sector. Throughout this paper, we take this specialization pattern as exogenously given, and hence it is not influenced by any changes in trade costs. Each country has a few number of identical producers with strategic interactions who are not of measure zero and affect market variables in each sector. Specifically, Foreign has $N(\geq 1)$ suppliers that produce and export the intermediate good, while Home has $M(\geq 1)$ firms that import the Foreign input and produce the final good. Thus, we consider *vertical oligopoly* in an international setting, whereby producers with strategic interactions export and import the intermediate good in the vertically related sectors.

[5] The constant-elasticity demand is frequently used not only in monopolistic competition but also in oligopoly in the literature. For example, Brander and Krugman (1983) employ it as a useful special case for the analysis of reciprocal dumping. More recently, Mrázová and Neary (2017) consider the role of general demands that permit the broader combinations of (ε, ρ) than (2).

Home firms and Foreign suppliers must first decide whether or not to incur entry costs K_H, K_F, respectively. Upon entry, they compete for quantity in the respective sectors. As in the previous work on vertical oligopoly (Ishikawa and Spencer, 1999; Ghosh and Morita, 2007), the analysis is simplified by assuming that Home firm i requires just one unit of the intermediate good to produce one unit of the final good q_i and costlessly transform it into the final good. Home firms import Foreign input subject to ad-valorem tariffs τ set by a Home government on the input price of r. Thus Home firm i's per unit costs are $r\tau$ and its profits are $\pi_{Hi} = (P - r\tau)q_i$ where the aggregate quantity of Home firms is $Q = \sum_i q_i$. In contrast, Foreign supplier i incurs marginal costs c to produce the intermediate good x_i and pays transport costs t to export it to Home firms. We treat t as iceberg trade costs in terms of the intermediate good that melts away on the way. For simplicity, a Foreign government does not set any trade policy. Thus Foreign supplier i's per unit costs are ct and its profits are $\pi_{Fi} = (r - ct)x_i$ where the aggregate quantity of Foreign suppliers is $X = \sum_i x_i$.

From the specialization pattern and trade policy described above, it is possible to define the terms-of-trade – relative price of exports to imports – in Home, which are crucial for understanding the welfare effect of tariffs in vertical oligopoly. Home imports the intermediate good from Foreign at the price of $r\tau$, while Home exports the freely-tradable numeraire to Foreign at the price of 1, where we assume, as is standard in the trade literature, that trade balance is achieved by this good. Taken together, the terms-of-trade in Home (net of tariffs) are given by $1/r$. As in horizontal specialization, the terms-of-trade motive is the main reason why the Home government sets tariffs on the Foreign input in vertical specialization. Further, the Home welfare is defined as

$$W \equiv \underbrace{\left[\int_0^Q P(y)dy - P(Q)Q\right]}_{\text{Consumer surplus}} + \underbrace{\Big[\big(P(Q) - r\tau\big)Q - MK_H\Big]}_{\text{Profits}} + \underbrace{\frac{r(\tau-1)X}{t}}_{\text{Tariff revenues}} + \underbrace{L}_{\text{Labor income}}, \qquad (3)$$

where profits are positive in a fixed number of firms and zero under free entry. Regardless of market structure, if tariffs lower the input price, this decreases the production costs of Home firms and benefits Home consumers, because the final-good production by Home firms rests on the Foreign input in our model. Thus, a decline in the input price by tariffs can be thought of as the terms-of-trade improvement.

As explained in the Introduction, this paper explores the role of trade policy in market-based interactions between vertically related sectors. For this reason, we assume that the input price is determined at the market-clearing level of the intermediate good, which equates the total input demand of Home firms to the total input supply of Foreign suppliers in the intermediate-good market. Taking the input price and the tariff rate as given, Home firms decide on their quantity of the final good as Cournot competitors. This price-taking behavior of Home firms implies that each Home firm has no oligopsony power over Foreign suppliers.[6]

The model has three stages of decisions. In Stage 0, the Home government sets tariffs on the Foreign input. In Stage 1, a large number of entrants decide on their entry in the downstream sector in Home and the upstream sector in Foreign. Should a Home firm decide to enter the downstream sector, it must incur the entry costs K_H. Similarly, a Foreign supplier incurs the entry costs K_F if it decides to enter the upstream sector. Entry of these producers occurs until any entrant makes zero profits. In Stage 2, Home firms and Foreign suppliers decide on their quantity in the downstream and upstream sectors, respectively. It is worth stressing that Home firms and Foreign suppliers simultaneously make the entry decisions in Stage 1, and the quantity decisions in Stage 2. Then one-to-one production technology means that the aggregate quantity Home firms choose is the same as that Foreign suppliers choose subject to melting transport costs in Stage 2, i.e., $Q = X/t$. To summarize,

[6] This assumption again follows from the literature on vertical oligopoly (Ishikawa and Spencer, 1999; Ghosh and Morita, 2007).

Stage 0: The Home government imposes import tariffs on the Foreign input.

Stage 1: Home firms and Foreign suppliers simultaneously decide on their entry in the respective sectors.

Stage 2: Home firms and Foreign suppliers simultaneously decide on their quantity in the respective sectors.

As usual, we solve the model backwards. The equilibrium concept we rely on in this paper is subgame perfect Nash equilibrium in pure strategies.

2.2 Exogenous Market Structure

As discussed in Section 2.1, Home firms in the downstream sector and Foreign suppliers in the upstream sector simultaneously decide on their quantity in Stage 2. Because of that, the sequence of production does not matter for equilibrium characterizations. In what follows, we first consider the equilibrium in the downstream sector, and then turn to analyzing the same in the upstream sector.

Each Home firm competes for quantity in the downstream sector, taking as given other Home firms' quantity, the input price determined in the intermediate-good market, and the tariff rate set by the Home government. Thus, Home firm i chooses its quantity q_i to maximize its profits $\pi_{Hi} = \big(P\big(q_i + \sum_{j \neq i}^{M} q_j\big) - r\tau\big)q_i$ in this sector. Solving standard profit-maximization problem of Home firms, we find a unique symmetric Nash equilibrium $q_1 = q_2 = ... = q_M \equiv q (> 0)$ which must satisfy the following:

$$q = -\frac{P(Q) - r\tau}{P'(Q)},$$

where the aggregate quantity in the downstream sector is obtained by summing q over M Home firms, $Q = Mq$. Multiplying q in the above equation by M and rearranging, the aggregate quantity is implicitly given by

$$MP(Q) + QP'(Q) = Mr\tau.$$

While the expression of aggregate quantity holds under general demands, we can simplify this expression under the constant-elasticity demand in (1). Dividing both sides by $P(Q)$, the second term in the left-hand side above is an inverse of demand elasticity $-\sigma$. Further, solving ρ for σ in (2), we get $\sigma = -1/(1+\rho)$. Taken together, under the specific demand, the aggregate quantity must implicitly satisfy the following:

$$M + 1 + \rho = \frac{Mr\tau}{P(Q)}. \tag{4}$$

(4) is the relationship between the input price r and the aggregate quantity $Q = X/t$ in vertical specialization. To see this point, inspection of (4) shows that, for a given tariff rate (τ), the smaller the input price (r) that Home firms pay, the greater the aggregate quantity that Home firms choose (Q) in the downstream sector. With input-output linkages, this increase means that the aggregate quantity that Foreign suppliers choose (X) is also greater in the upstream sector, because the Foreign input is more demanded by Home firms. Recalling that the input price is determined at the market-clearing level, (4) defines the demand for intermediate input required by Home firms and perceived by Foreign suppliers in the upstream sector. Using $Q = X/t$ for (4) and rearranging, the inverse demand function for input is given by

$$r = \frac{P(X/t)(M + 1 + \rho)}{M\tau} \equiv g(X, M, \tau, t). \tag{5}$$

(5) shows that the input price depends on: (i) aggregate quantity produced by Foreign suppliers; (ii) number of Home firms; (iii) import tariffs paid by Home firms; and (iv) transport costs paid by Foreign suppliers. Treating the number of Home firms as a continuous variable, (5) implies that the input demand satisfies

$$\begin{aligned} g_X(.) &= \frac{P'(.)(M+1+\rho)}{M\tau t} < 0, \qquad g_M(.) = -\frac{P(.)(1+\rho)}{M^2\tau} > 0, \\ g_\tau(.) &= -\frac{P(.)(M+1+\rho)}{M\tau^2} < 0, \qquad g_t(.) = -\frac{XP'(.)(M+1+\rho)}{M\tau t^2} > 0, \end{aligned} \tag{6}$$

where the signs in (6) come from the demand curvature ranging $-2 < \rho < -1$ from (2). Then, (6) shows that: (i) the input demand is downward-sloping; (ii) an increase in the number of Home firms raises the input demand; (iii) an increase in import tariffs raises the production costs of Home firms, declining the input demand; and (iv) an increase in transport costs raises the production costs of Foreign suppliers, declining the input supply. Note that the two kinds of variable trade costs have the opposite impact because (5) represents the equilibrium input price determined at the market-clearing level, equaling the demand and supply of the Foreign input.

The following lemma records the relationship between the final-good demand $P = P(.)$ in (1) and the input demand $r = g(.)$ in (5).

Lemma 1: *The elasticity and curvature of demand are the same between the final and intermediate goods.*

Proof. From (5) and (6), the elasticity and curvature of input demand are, respectively, given by

$$\begin{aligned} -\frac{g(X,M,\tau,t)}{Xg_X(X,M,\tau,t)} &= -\frac{P(Q)}{QP'(Q)} = \varepsilon, \\ \frac{Xg_{XX}(X,M,\tau,t)}{g_X(X,M,\tau,t)} &= \frac{QP''(Q)}{P'(Q)} = \rho, \end{aligned} \tag{7}$$

where the first equalities in (7) follow from noting (5), (6) and $g_{XX}(.) = \frac{P''(.)(M+1+\rho)}{M\tau t^2}$. □

Lemma 1 shows that the elasticity and curvature of demand are exactly the same between the final good and the intermediate good, so long as both measures are assumed to be constant. As a consequence, the elasticity and curvature of demand in (2) applies not only to the final-good demand (of Home consumers) but also to the intermediate-good demand (of Home firms). Admittedly, this feature of demand structure is very special and holds only under constant-elasticity demand, but it helps to simplify the characterization of vertical oligopoly. This is the main reason why we decide to use the specific demand in this paper.[7]

We turn to Foreign suppliers' behaviors. Each Foreign supplier competes for quantity in the upstream sector, taking other Foreign suppliers' quantity as given and perceiving the input demand of Home firms shown by (5). Thus, Foreign supplier i chooses its quantity x_i to maximize its profits $\pi_{Fi} = \big(g\big(x_i + \sum_{j\neq i}^N x_j, M, \tau, t\big) - ct\big)x_i$. Solving standard profit-maximization problem of Foreign suppliers yields a unique symmetric Nash equilibrium $x_1 = x_2 = ... = x_N \equiv x (> 0)$ which must satisfy the following:

$$x = -\frac{g(X,M,\tau,t) - ct}{g_X(X,M,\tau,t)},$$

[7] In a discussion paper version of the present study (Ara et al., 2024), we have worked with more general demands which yield more realistic results, but it comes at the expense of complexity. The equivalence of demand measures in (7) would also depend on the one-to-one production structure, though it is quite common in the vertical oligopoly literature (Lemma 1 would not hold with more general technology even with constant-elasticity demand).

where the aggregate quantity $X = Nx$ is implicitly given by

$$Ng(X, M, \tau, t) + Xg_X(X, M, \tau, t) = Nct.$$

As before, an implicit solution to the aggregate quantity can be simplified under the constant-elasticity demand. Dividing both sides by $g(.)$ and using (2) and (7), the above condition is expressed as

$$N + 1 + \rho = \frac{Nct}{g(X, M, \tau, t)}. \tag{8}$$

In general models, the first-order conditions in (4) and (8) separately characterize the aggregate quantity in each sector. In the present model, however, (8) suffices to characterize the aggregate quantity in both sectors. More specifically, (8) first characterizes X in the upstream sector, which in turn characterizes $Q = X/t$ in the downstream sector. Exploiting it, the following lemma reports the comparative statics results with respect to variable trade costs (τ, t) and numbers of producers (M, N) in the Stage-2 subgame equilibrium.

Lemma 2: *When the market structure in each production sector is exogenously fixed, we have the following effects on equilibrium variables.*[8]

(*i*) *An increase in import tariffs and transport costs decreases the aggregate and average quantities in the vertically related sectors. In case of import tariffs, we have*

$$\frac{\partial Q}{\partial \tau} < 0, \; \frac{\partial X}{\partial \tau} < 0, \; \frac{\partial q}{\partial \tau} < 0, \; \frac{\partial x}{\partial \tau} < 0.$$

(*ii*) *An increase in numbers of producers increases the aggregate quantity, but decreases the average quantity between the vertically related sectors. In case of the number of Home firms, we have*

$$\frac{\partial Q}{\partial M} > 0, \; \frac{\partial X}{\partial M} > 0, \; \frac{\partial q}{\partial M} < 0, \; \frac{\partial x}{\partial M} > 0.$$

(*iii*) *Import tariffs have no effect on the terms-of-trade for Home.*

$$\frac{\partial r}{\partial \tau} = 0.$$

Proof. Lemma 2(i)-(ii) follow from differentiating (8) and using (6) where $Q = X/t$. In case of import tariffs,

$$\frac{\partial Q}{\partial \tau} = \frac{1}{1+\rho}\frac{Q}{\tau} < 0. \tag{9}$$

On the other hand, Lemma 2(iii) follows from taking the partial derivative of $r = g(.)$ in (5) with respect to τ. Using (6) and $\frac{\partial X}{\partial \tau} = \frac{1}{1+\rho}\frac{tQ}{\tau}$ from (9), this derivative yields

$$\frac{\partial r}{\partial \tau} = g_X(.)\frac{\partial X}{\partial \tau} + g_\tau(.) = \frac{P(.)(M+1+\rho)}{M\tau^2}\left(\frac{QP'(.)}{P(.)}\frac{1}{1+\rho} - 1\right) = 0,$$

where the last equality comes from (2). ☐

[8] Appendix A.1 provides detailed expressions of partial derivatives in this lemma, including those of transport costs.

Unsurprisingly, the aggregate quantity is decreasing in trade costs but is increasing in numbers of producers. An increase in trade costs acts as increased production costs, whereas an increase in numbers of producers acts as increased competition. An increase in trade costs also decreases the average quantity ("intensive margin"). In contrast, an increase in numbers of producers has nuanced effects on the intensive margin. Entry in one sector steals business from incumbent producers in that sector but it creates business in another sector due to input-output linkages. For example, if the number of Home firms rises, this leads to increased competition in the downstream sector and reduces Home firms' intensive margin ($\frac{\partial q}{\partial M} < 0$); however, this also leads to increased demand for intermediate input in the upstream sector and raises Foreign suppliers' intensive margin ($\frac{\partial x}{\partial M} > 0$). The same is true for the number of Foreign suppliers.

Since an increase in tariffs decreases the aggregate quantity, the increased elasticity of demand usually lowers the input price. Under general demands, this outcome occurs, which reflects that the Home government sets tariffs on the Foreign input to improve the terms-of-trade, knowing that Home production relies on such input in vertical specialization (recall that the terms-of-trade are defined as the inverse of input price in our model).[9] Under constant-elasticity demand, however, the channel is not operative because the elasticity of demand is constant for both final and intermediate goods (Lemma 1). This means that the decreased aggregate quantity by an increase in tariffs does not affect the elasticity of input demand and, therefore, the input price as well.[10] Although the result is unsatisfactory, the specific demand greatly helps to maintain our analysis tractable and provide precise characterizations of the impact of tariffs in vertically related sectors.

2.3 Endogenous Market Structure

Now, we consider the Stage-1 subgame where the market structure is endogenously determined via free entry. In this stage, expecting the aggregate quantities Q, X and the goods prices P, r that are determined in Stage 2, each Home firm incurs the fixed costs K_H to enter the downstream sector, whereas each Foreign supplier incurs the fixed costs K_F to enter the upstream sector. For simplicity, we assume that these fixed costs are so small that at least one Home firm and one Foreign supplier enter the respective sectors.

Before analyzing the entry decision of producers in the vertically related sectors, we first need to stress that Home firms and Foreign suppliers simultaneously make their entry decisions in Stage 1, as seen in Section 2.1, and hence the sequence of entry is irrelevant to equilibrium characterizations. From this reason, we consider the entry decisions of Home firms and Foreign suppliers at the same time. Let $\pi_H \equiv (P - r\tau)q,\ \pi_F \equiv (r - ct)x$ denote the post-entry profits of Home firms and Foreign suppliers, respectively, where the quantities and prices of final and intermediate goods are determined in Stage 2. In the Stage-1 subgame equilibrium, entry occurs until the post-entry profits equal the entry costs K_H, K_F so that the net profits are zero: $\pi_H = K_H$, $\pi_F = K_F$. Under free entry, the numbers $M,\ N$ are consistent with zero profits for Home firms and Foreign suppliers, respectively. Moreover, in the symmetric Nash equilibrium, the average quantities satisfy $q = Q/M,\ x = X/N$ for Home firms and Foreign suppliers, respectively. Using the zero profits in M, N and the relationships in q, x, we obtain the free entry conditions in the vertically related sectors:

$$\frac{(P - r\tau)Q}{M} = K_H, \tag{10}$$

$$\frac{(r - ct)X}{N} = K_F. \tag{11}$$

[9]Using more general demands, Ara et al. (2024) show that this government motive actually works in the present setting.

[10]This is similar to a pricing rule of firms under constant-elasticity demand: the constant price elasticity of demand means that changes in competing firms' price or quantity does not affect any firms' price.

The equilibrium number of Home firms M and that of Foreign suppliers N are implicitly given as a solution to (10) and (11) in the downstream and upstream sectors, respectively. In this manner, the number of producers M, N ("extensive margin") are endogenously determined via free entry in the Stage-1 subgame equilibrium of our vertical oligopoly model.

We have shown that the aggregate quantities Q, X are solely determined by the first-order condition in (8). From these quantities in Stage 2, the numbers of producers M, N are then determined to satisfy the free entry conditions in (10) and (11), which in turn determines the average quantities $q = Q/M,\ x = X/N$ in Stage 1. Thus, (8), (10) and (11) characterize three unknowns $Q(= X/t), M$ and N in the Stage-1 subgame equilibrium. Solving these equilibrium conditions simultaneously, we are able to address how the trade costs (τ, t) affect the aggregate quantities (Q, X), the numbers of producers (M, N) and the average quantities (q, x) in each sector. The following proposition summarizes this impact in the Stage-1 subgame equilibrium.

Proposition 1: *When the market structure in each production sector is endogenously determined via free entry, we have the following effects on equilibrium variables.*[11]

(i) *An increase in import tariffs and transport costs decreases not only the aggregate and average quantities but also the numbers of producers in the vertically related sectors. In case of import tariffs, we have*

$$\frac{dQ}{d\tau} < 0,\ \frac{dX}{d\tau} < 0,\ \frac{dM}{d\tau} < 0,\ \frac{dN}{d\tau} < 0,\ \frac{dq}{d\tau} < 0,\ \frac{dx}{d\tau} < 0.$$

(ii) *Import tariffs worsen the terms-of-trade for Home.*

$$\frac{dr}{d\tau} > 0.$$

Proof. Consider the effect of import tariffs given in Proposition 1(i). Taking the log and totally differentiating (8), (10) and (11) with respect to τ where $Q = X/t$, we get

$$\begin{aligned}
&(1+\rho)\hat{Q} = \left(\frac{1+\rho}{M+1+\rho}\right)\hat{M} + \left(\frac{1+\rho}{N+1+\rho}\right)\hat{N} + \hat{\tau},\\
&(2+\rho)\hat{Q} = 2\hat{M},\\
&(2+\rho)\hat{Q} = \left(\frac{1+\rho}{M+1+\rho}\right)\hat{M} + 2\hat{N} + \hat{\tau},
\end{aligned}$$

where a "hat" denotes a proportional change in a variable, e.g., $\hat{Q} \equiv dQ/Q$. Then the result in Proposition 1(i) follows from noting that this is the system of three equations with three unknowns $(\hat{Q}, \hat{M}, \hat{N})$. Specifically,

$$\frac{dQ}{d\tau} = \frac{\mu(M,N,\rho)}{1+\rho}\frac{Q}{\tau} < 0, \tag{12}$$

where

$$\mu(M,N,\rho) \equiv \frac{2(M+1+\rho)(2N+1+\rho)}{(2M+\rho)(2N+\rho)-(2+\rho)}. \tag{13}$$

On the other hand, Proposition 1(ii) follows from rewriting the free entry condition in (11) as $r = ct + K_F/x$, totally differentiating this with respect to τ and using $dx/d\tau < 0$. □

[11]See Appendix A.2 for detailed derivations, showing the similar effects between import tariffs and transport costs.

The first part of Proposition 1 means that an increase in variable trade costs affects the aggregate quantity not only by increasing the production costs, but also by decreasing the number of producers (extensive margin) in each sector. For example, the effect of tariffs in the downstream sector is decomposed as

$$\frac{dQ}{d\tau} = \underbrace{\frac{\partial Q}{\partial \tau}}_{\text{Direct effect}} + \underbrace{\frac{\partial Q}{\partial M}\frac{dM}{d\tau} + \frac{\partial Q}{\partial N}\frac{dN}{d\tau}}_{\text{Indirect effects from co-movement}}. \tag{14}$$

In (14), the first term is a direct effect which exists even when the number of producers is fixed as in (9), while the second and third terms are indirect effects which exist only when the number of producers is endogenous. Lemma 2 shows that the aggregate quantity is increasing in the numbers of producers ($\frac{\partial Q}{\partial M} > 0$, $\frac{\partial Q}{\partial N} > 0$). Moreover, an increase in tariffs increases the production costs of Home firms and decreases the input demand perceived by Foreign suppliers, forcing incumbent producers to exit the respective sectors ($\frac{dM}{d\tau} < 0$, $\frac{dN}{d\tau} < 0$). As a result of these, an increase in tariffs decreases the final good produced by Home firms ($\frac{\partial Q}{\partial M}\frac{dM}{d\tau} < 0$) and decreases the intermediate good produced by Foreign suppliers through the input-output linkages ($\frac{\partial X}{\partial N}\frac{dN}{d\tau} < 0$). The same holds for the aggregate quantity in the upstream sector from $X = tQ$.

As is clear from the above explanation, the amplified effect of tariffs is unique to our *vertical* oligopoly model where traded goods are *complements*: an increase in tariffs decreases the aggregate quantity in both countries, since this forces incumbent Home firms to exit (by increasing their production costs) in the downstream sector and forces incumbent Foreign suppliers to exit (by losing their business) in the upstream sector. In this sense, tariffs lead to *co-movement* of producers in the vertically related sectors. This differs from *horizontal* oligopoly models where traded goods are *substitutes*: an increase in tariffs leads to *de-location* among trading countries, i.e., Home has more entry of firms (by softening Foreign competition) but Foreign has less entry (by limiting the access to the Home market). The shift in entry, in turn, generates opposite welfare effects between countries.[12] We will elaborate on this long-run welfare effect of tariffs in Section 2.4.

In contrast to this, an increase in variable trade costs can either increase or decrease the average quantity. Consider the effect on Home firms' intensive margin. From $q = Q/M$ where both Q and M decrease with τ, an increase in tariffs can increase the aggregate quantity more or less proportionately than the number of firms. The ambiguity is also understood by decomposing this effect as

$$\frac{dq}{d\tau} = \underbrace{\frac{\partial q}{\partial \tau}}_{\text{Direct effect}} + \underbrace{\frac{\partial q}{\partial M}\frac{dM}{d\tau} + \frac{\partial q}{\partial N}\frac{dN}{d\tau}}_{\text{Indirect effects from co-movement}}. \tag{15}$$

Lemma 2 shows that an increase in tariffs decreases the intensive margin for a given number of firms ($\frac{\partial q}{\partial \tau} < 0$), but it also decreases the extensive margin. As incumbent Foreign suppliers exit, the input price gets higher, which leads each Home firm to decrease their quantity ($\frac{\partial q}{\partial N}\frac{dN}{d\tau} < 0$). As incumbent Home firms exit, however, each Home firm can exploit the scale of production, which increases their quantity ($\frac{\partial q}{\partial M}\frac{dM}{d\tau} > 0$). Owning to these additional forces that work in opposite directions, the effect of tariffs on the intensive margin is generally ambiguous. Under constant-elasticity demand, the indirect effects are necessarily smaller than the direct effect, because the elasticity of demand is constant in both final and intermediate goods (Lemma 1) and the extent to which competition is weakened (due to the exit of incumbent firms in the downstream sector) is limited. Thus, an increase in tariffs always decreases Home firms' intensive margin under the specific demand. The same holds for Foreign suppliers' intensive margin from $x = X/N$.

[12]For example, Bagwell and Staiger (2012) use the Cournot de-location model to consider this kind of opposite welfare effects.

To understand the second part of Proposition 1, decompose the effect of tariffs on the input price as

$$\frac{dr}{d\tau} = \underbrace{\frac{\partial r}{\partial \tau}}_{\text{Direct effect}} + \underbrace{g_X(X, M, \tau, t)\frac{\partial X}{\partial N}\frac{dN}{d\tau}}_{\text{Indirect effect from co-movement}}. \quad ^{13} \tag{16}$$

An increase in tariffs decreases the input demand of Home firms by increasing their production costs and lowers the market-clearing input price, which acts towards improving the terms-of-trade for Home. This direct effect is reflected in the first term of (16); however, this channel is not operative under the constant-elasticity demand, as argued in Lemma 2. With free entry, an increase in tariffs also affects the input price from a different channel: the decreased input demand of Home firms (by increased tariffs) forces incumbent Foreign suppliers to exit and decrease the aggregate quantity in the upstream sector. The increased input price acts towards worsening the terms-of-trade for Home. This indirect effect is reflected in the second term of (16), which always arises under any demand structure. Taken together, so long as we consider constant-elasticity demand, an increase in tariffs worsens the terms-of-trade. Since $\frac{\partial r}{\partial \tau} > 0$ under non-constant-elasticity demand, the result does not necessarily hold in more general settings. Our model implies, however, that input-output linkages across nations make it more difficult to realize the terms-of-trade improvement by tariffs in the long-run equilibrium, due mainly to the co-movement of firms and suppliers in vertical specialization.

We have shown that the co-movement of producers associated with entry-exit considerations might change the impact of variable trade costs on key equilibrium variables.[14] This result suggests that the indirect effects occur only when there are a few number of firms and suppliers in the vertically related sectors. In other words, when the vertically related sectors are close to be perfectly competitive with hundreds of firms and suppliers, each producer plays a minor role in the respective sectors, where the co-movement has only a marginal effect. The following lemma summarizes this statement in a formal manner.

Lemma 3: *The indirect effects from co-movement of producers disappear when the vertically related sectors are perfectly competitive. In case of the impact of tariffs on the aggregate quantity in Home, we have*

$$\lim_{M\to\infty, N\to\infty} \frac{dQ}{d\tau} = \frac{\partial Q}{\partial \tau}.$$

Proof. Simple inspection of (13) immediately reveals that

$$\lim_{M\to\infty, N\to\infty} \mu(M, N, \rho) = 1.$$

The result follows from comparing (9) and (12). Moreover, (14) shows that this result holds if and only if the indirect effects are zero. Since competitive firms have no effect on aggregate quantity, it follows from (14) that

$$\lim_{M\to\infty, N\to\infty} \frac{dQ}{dM} = \frac{dQ}{dN} = 0.$$

The same result holds for (15) and (16), in the sense that the indirect effects from co-movement are zero when the vertically related sectors are perfectly competitive. □

[13] From $r = g(X, M, \tau, t)$, tariffs generally affect the input price from the number of Home firms M. However, the dependence is eliminated under constant-elasticity demand (see Appendix A.2).

[14] For expositional purposes, we have focused only on the impact of import tariffs on the key equilibrium variables above; however, qualitatively similar results hold for the effect of transport costs (see Appendix A.4).

It is worth stressing that Lemma 3 holds only when *both* vertically related sectors are perfectly competitive. Indeed, (13) shows that $\mu(.)$ is greater than 1 when one of the vertically related sectors is oligopolistic (keeping another sector perfectly competitive). Further, (14) shows that one of the indirect effects remains in that case. This highlights the role of vertical oligopoly in this equivalence result.

We are now ready to calculate the elasticity of total input imports with respect to variable trade costs – often referred to as the "trade elasticity" in the literature – in the present paper.

Proposition 2: *The trade elasticities with respect to import tariffs and transport costs critically depend upon the degree of oligopolistic competition in the vertically related sectors. Specifically, noticing that import tariffs (transport costs) are incurred by Home firms (Foreign suppliers) in the downstream (upstream) sector, we get*

$$\begin{aligned} \theta &\equiv -\frac{d\ln Q}{d\ln \tau} = \sigma\mu(M,N,\rho), \\ \vartheta &\equiv -\frac{d\ln X}{d\ln t} = (\sigma-1)\eta(M,N,\rho), \end{aligned} \tag{17}$$

where $\mu(.)$ is given in (13) and $\eta(.)$ has the same properties as $\mu(.)$ stated below:

(i) $\mu(.) = \eta(.) = 1$ *for* $M = N = \infty$*. The trade elasticities are constant when the vertically related sectors are perfectly competitive.*

(ii) $\mu(.) > 1$, $\eta(.) > 1$ *for* $M, N < \infty$*. The trade elasticities are variable and strictly decreasing in* M, N *when the vertically related sectors are oligopolistic.*

Proof. The trade elasticity with respect to import tariffs is obtained by plugging $\sigma = -1/(1+\rho)$ in (12) and noting the definition of θ in (17), whereas its properties are obtained by noting the definition of $\mu(.)$ in (13). The same proof applies to the trade elasticity with respect to transport costs (see Appendix A.2 for the exact expression of $\eta(.)$). □

Proposition 2 means that the trade elasticities with respect to variable trade costs are larger in the vertically related sectors with the smaller numbers of producers. The intuition of θ is explained as follows. When a huge number of producers exist in the sectors, the indirect effect of co-movement has no effect on aggregate quantity (Lemma 3). Thus, the trade elasticity with respect to tariffs collapses to the standard one, σ, without strategic interactions, which arises in perfect or monopolistic competition under constant-elasticity demand, i.e., $\theta = \sigma$. In contrast, when only a few number of producers exist in the sectors, each producer is not of measure zero and affects market variables in each sector. Specifically, the co-movement of producers amplifies the effect of tariffs on aggregate quantity, as reflected by the second term of (14). Thus, the trade elasticity with respect to tariffs is not constant even under constant-elasticity demand and is larger than the standard one, σ, for $M, N < \infty$. While $\theta = \sigma$ only when both sectors are perfectly competitive, $\theta > \sigma$ even when one of the sectors is oligopolistic (because $\mu(.) > 1$ in such a case), retaining the variable nature of our trade elasticity stated in Proposition 2. The same result holds for the trade elasticity with respect to transport costs.[15]

[15] While the present study defines the trade elasticities in terms of *quantity*, previous studies often define them in terms of *value*; however, different measures of trade elasticities have similar properties, so long as total imports are valued at the fob price, i.e., the price excluding import tariffs and transport costs. For example, the fob aggregate import value (net of tariffs) is $R \equiv PQ/\tau$, which yields $\tilde{\theta} \equiv -\frac{d\ln R}{d\ln \tau} = (\sigma-1)\mu(.) + 1$. Thus, the results still hold in that $\tilde{\theta} = \sigma$ for $M = N = \infty$ and $\tilde{\theta} > \sigma$ for $M, N < \infty$. This approach is also at the core to estimate the tariff-based product-level trade elasticity, as shown by Fontagné et al. (2022).

As is widely known, the trade elasticity is one of the sufficient statistics for welfare (Arkolakis et al., 2012). To the best of our knowledge, however, previous work has mainly derived the trade elasticity under conditions of perfect competition in the multistage production (Antràs and de Gortari, 2020; Johnson and Moxnes, 2023). The present model is partial equilibrium and thus the Arkolakis et al. (2012) formula is not directly applicable; however, the novel feature of our trade elasticity calls for a welfare estimation unique to vertical specialization where a few number of oligopolistic producers dominate trade flows. Based on this view, we turn to examining the welfare effect of tariffs by paying attention to the role of free entry in vertical specialization.

2.4 Welfare Effect of Tariffs

Now consider Stage 0 where the Home government sets import tariffs on the Foreign input. As shown by (3), the Home welfare consists of consumer surplus, profits, tariff revenues and labor income in our model. Using $Q = X/t$ and rearranging, we can express the Home welfare in terms of the aggregate quantity in Home only.[16] While the equilibrium variables in the Home welfare are determined in Stage 1, compare the welfare effect of tariffs between the equilibria of a fixed and endogenous number of producers via free entry, with the difference being whether or not there is co-movement in the vertically related sectors. Our key interest lies in explaining how the effect of tariffs generates different consequences for welfare between these two different environments. The following proposition summarizes the welfare result of tariffs in the Stage-0 subgame equilibrium.

Proposition 3: *The welfare losses of tariffs are larger in an endogenous number of producers via free entry than those in a fixed number of producers.*

Proof. Differentiating W with respect to τ and using $\frac{dQ}{d\tau} = q\frac{dM}{d\tau} + M\frac{dq}{d\tau}$,

$$\frac{dW}{d\tau} = \Big((P - r\tau)q - K_H\Big)\frac{dM}{d\tau} + r(\tau - 1)\frac{dQ}{d\tau} + M(P - r\tau)\frac{dq}{d\tau} - Q\frac{dr}{d\tau}.$$

Note that the first term is zero in a fixed number of firms (from $\frac{dM}{d\tau} = 0$) and in an endogenous number of firms (from $(P - r\tau)q - K_H = 0$ in (10)) in the downstream sector. Further, $\frac{dQ}{d\tau}$, $\frac{dq}{d\tau}$ and $\frac{dr}{d\tau}$ in the remaining terms satisfy (14), (15) and (16), respectively, where the indirect effects are either absent or present.

When the number of producers is fixed, the indirect effects from co-movement are absent. This means that $\frac{dQ}{d\tau} = \frac{\partial Q}{\partial \tau}$, $\frac{dq}{d\tau} = \frac{\partial q}{\partial \tau}$ and $\frac{dr}{d\tau} = 0$ in (14), (15) and (16), respectively. Using these for the welfare changes above, the welfare effect of tariffs in the exogenous market structure is given by

$$\frac{\partial W}{\partial \tau} = r(\tau - 1)\frac{\partial Q}{\partial \tau} + M(P - r\tau)\frac{\partial q}{\partial \tau} < 0, \tag{18}$$

where the sign of (18) follows from Lemma 2.

When the number of producers is endogenous via free entry, the indirect effects of co-movement are present. Taking into account the additional channels in (14), (15) and (16) together, the welfare effect of tariffs in the endogenous market structure via free entry is given by

$$\frac{dW}{d\tau} = r(\tau - 1)\left(\frac{\partial Q}{\partial \tau} + \frac{\partial Q}{\partial M}\frac{dM}{d\tau} + \frac{\partial Q}{\partial N}\frac{dN}{d\tau}\right) + M(P - r\tau)\left(\frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial M}\frac{dM}{d\tau} + \frac{\partial q}{\partial N}\frac{dN}{d\tau}\right) - Q\frac{dr}{d\tau} < 0, \tag{19}$$

[16] Specifically, $W = \int_0^Q P(y)dy - r\tau Q - MK_H + r(\tau - 1)Q + L$.

where the sign of (19) follows from Proposition 1. Observe that increased tariffs worsen Home's terms-of-trade ($\frac{dr}{d\tau} > 0$), which negatively affects the Home welfare in this case.

Finally, subtracting (18) from (19), we get

$$\frac{dW}{d\tau} - \frac{\partial W}{\partial \tau} = (P - r)\left(\frac{\partial Q}{\partial M}\frac{dM}{d\tau} + \frac{\partial Q}{\partial N}\frac{dN}{d\tau}\right) - Q\frac{dr}{d\tau} < 0. \tag{20}$$

The result in Proposition 3 follows from noting $\frac{\partial W}{\partial \tau} < 0$, $\frac{dW}{d\tau} < 0$ in (20). □

Proposition 3 suggests that, while increased tariffs always generate the welfare losses in both (18) and (19), these losses are always greater in an endogenous number of producers than in a fixed number of producers ($\left|\frac{dW}{d\tau}\right| > \left|\frac{\partial W}{\partial \tau}\right|$). To obtain the intuition, note that there are two key differences between these two scenarios, captured by the first and second terms in (20). The first term is the "anti-competitive effect," in the sense that the exit of producers (due to increased tariffs) decreases the aggregate quantity, exacerbating the welfare losses from tariffs. Indeed, totally differentiating (4) with respect to τ, we have the following pass-through of tariffs on the relative profit margin of Home firms:

$$\frac{d\ln P}{d\ln\tau} - \frac{d\ln(r\tau)}{d\ln\tau} = \left(\frac{1+\rho}{M+1+\rho}\right)\frac{d\ln M}{d\ln\tau} > 0.$$ [17]

Thus, increased tariffs give rise to the larger profit margin and weaker competition in the downstream sector, which is absent in a fixed number of firms. On the other hand, the second term is the "terms-of-trade effect," in the sense that the exit of producers raises the Foreign input price, exacerbating the welfare losses from tariffs even further. We have shown that increased tariffs decreases the input demand of Home firms, which, in turn, loses Foreign suppliers' business and forces incumbent suppliers to exit the upstream sector (Proposition 1). This leads to the additional welfare losses by increasing the input price for Home firms and the final-good price for Home consumers.

It is worth mentioning that the anti-competitive effect arises among Foreign suppliers in the present model. From (8), the pass-through of tariffs on the relative profit margin of these suppliers is

$$\frac{d\ln r}{d\ln\tau} - \frac{d\ln(ct)}{d\ln\tau} = \left(\frac{1+\rho}{N+1+\rho}\right)\frac{d\ln N}{d\ln\tau} > 0.$$

Thus, increased tariffs also give rise to the larger profit margin and weaker competition in the upstream sector, where the anti-competition effect of tariffs transmits across all trading countries through input-output linkages. Since the effect relates to the tariff-induced exit of producers in each stage, this means that trade policy shocks have similar effects on each country involved in vertical specialization through a complementary force on the extensive margin, which stands in contrast to horizontal specialization of de-location, as noted in Section 2.3. This finding bears some resemblance to that in Antràs et al. (2025), but one of the crucial differences is that we focus on foreign outsourcing among a few number of oligopolistic producers.

We conclude our analysis by noting that the amplified effects of tariffs on welfare under free entry are absent when the vertically related sectors are perfectly competitive. In this special case, it follows from Lemma 3 that the indirect effects from co-movement disappear and thus the difference in welfare losses in (20) reduces to zero between the two scenarios ($\left|\frac{dW}{d\tau}\right| = \left|\frac{\partial W}{\partial \tau}\right|$).

[17] This effect of tariffs is known as the *relative* pass-through (Mrázová and Neary, 2017). In contrast, the *absolute* pass-through is defined as the effect of tariffs on the absolute profit margin, which also occurs in our model. See Appendix A.3 for details.

3 Empirical Analysis

In this section, we empirically examine the relationship between import tariffs and intermediate goods, with a particular focus on market concentration. Specifically, we use detailed Chinese Customs data and product-level import tariff rates. Our findings suggest that products with higher market concentration are more likely to be affected by import tariffs.

3.1 Data

Our import data are derived from the census of export and import transactions of Chinese firms from 2000 to 2007, collected by China's General Administration of Customs. The database records a firm's trade value, import source, export destination, and trade mode (processing trade or ordinary trade) at the 8-digit Harmonized System (HS) product level. The Chinese Customs data have been widely examined in the previous studies on the trade activities and performance of Chinese firms (Ahn et al., 2011; Brandt et al., 2017). The data report free-on-board (FOB) values for both exports and imports in thousands of U.S. dollars. They also include the quantities traded, measured in one of 12 different units (e.g., kilograms, square meters), enabling us to examine the effect of tariffs on import quantities.

For the purpose of this empirical study, we construct the dataset in four steps, as described below. First, in the Customs data, some firms are pure trading companies that do not engage in manufacturing activities. Following standard practice in the literature (Ahn et al., 2011), we identify such intermediaries and wholesalers using keywords in firm names and excluded them from our sample. Imports handled by trading companies accounted for a significant but declining share of total imports over the sample period in the data. We focus on manufacturing firms that use imported intermediate goods in their production.

Second, we exclude imports entering China under processing trade arrangements, which are exempt from import duties and account for approximately half of total imports. Instead, we focus on intermediate goods under ordinary trade. Since our interest lies in the effect of tariffs on input trade within vertical specialization, processing trade is not appropriate for our analysis.

Third, since our theoretical framework focuses on vertical specialization in which the home country engages in downstream production using imported intermediate goods, we use the Broad Economic Categories (BEC) classification to identify imports as raw materials, intermediate goods, capital goods, or consumption goods. In this study, we exclude final goods, i.e., capital goods and consumption goods from the analysis using a concordance table between BEC classifications and 6-digit HS codes. We rely on publicly available concordance tables from UN Comtrade to ensure consistency in product codes over time.

Fourth, to measure market concentration, we calculate the Herfindahl-Hirschman Index (HHI) and the four-firm concentration ratio (CR4) at the 6-digit HS product level. We then aggregate import value, import quantity, and the number of importers at the product level. Import values are reported in thousands of U.S. dollars and converted into thousands of Chinese RMB using the annual exchange rate published by China's National Bureau of Statistics. Furthermore, we deflate the import values using input deflators constructed by Brandt et al. (2017).

Import tariff rates are obtained from the Trade Analysis Information System (TRAINS) database, available through the World Integrated Trade Solution (WITS). The dataset covers more than 5,000 HS 6-digit products and provides detailed information on tariff lines, including average, minimum, and maximum ad valorem tariff rates. We use the effectively applied tariffs imposed by China from 2000 to 2007 and map them to the Chinese Customs data to obtain product-level output tariffs for our analysis. On average, China's import tariff on

Table 1: Summary statistics

Variable	Obs	Mean	S.D.	Min	Max
log(Imports value)	28,127	7.24	2.98	-6.99	17.55
log(Import quantity)	28,127	11.54	4.18	0	25.53
Number of importers	28,127	147.59	392.47	1	9,298
Applied (output) tariff (%)	28,127	10.05	7.89	0	121.60
Input tariff (%)	28,127	7.06	2.90	1.92	26.85
HHI	28,127	0.30	0.27	0	1
High HHI dummy	28,127	0.46	0.50	0	1
CR4	28,127	0.74	0.25	0.3	1

Source: Authors' compilation based on Chinese Customs data, WITS, and Brandt et al. (2017).

intermediate goods declined from 14.2% in 2000 to 7.0% in 2007. It is important to note that in our empirical analysis, we restrict the sample to manufacturing firms and assume that these downstream importers use imported input from foreign countries in their production activities.

To explicitly incorporate input-output linkages into the current study, we use the input tariffs constructed by Brandt et al. (2017) as import tariffs. Input tariffs are calculated as weighted averages of output tariffs, with weights based on industry input shares from the 2002 Chinese Input-Output (IO) table. Due to the higher level of aggregation in the IO table, the input tariffs are effectively measured at the 3-digit level of China's Industrial Classification (CIC). By constructing a consistent industry-product mapping (IO-CIC-HS) over time, we obtain a product-level measure of import tariffs that is comparable across the period from 2000 to 2007.

Table 1 reports summary statistics for key variables in our empirical analysis. We find substantial variation across products in import value, import quantity, number of importers, tariffs, and market concentration. The average applied (output) tariff is 10.05%, while the average input tariff is 7.06%. On average, input tariffs are lower than applied (output) tariffs, which is consistent with the findings of Brandt et al. (2017). The High-HHI dummy takes the value of one if the HHI exceeds the median value of 0.20. Table A.2 in Appendix reports the correlation matrix of the key variables. Import value and quantity are negatively associated with both output and input tariffs, as well as with market concentration. The correlation coefficient between output tariffs and input tariffs is 0.52.

3.2 Specification

Our empirical specifications exploit cross-product variation in input tariffs. Focusing on market concentration, we estimate the heterogeneous effects of input tariffs on import value and quantity as follows:

$$Y_{pt} = \alpha + \beta_1 \log(tariff + 1)_{pt-1} + \beta_2 MC_{pt-1} + \beta_3 \log(tariff + 1)_{pt-1} * MC_{pt-1} + FE_p + FE_t + \varepsilon_{pt},$$

where Y_{pt} denotes the logarithm of the import value or import quantity of product p and year t. The term $\log(tariff + 1)_{pt-1}$ represents the one-year lagged input tariffs constructed using the input-output table. Since tariffs are expressed in percentage points (e.g., 10% ad-valorem tariffs recorded as 10), we divide the tariff rate by 100 and then apply a log transformation, $\log(tariff + 1)_{pt-1}$, to obtain a price-equivalent measure. MC_{pt-1} denotes market concentration, which is measured using the HHI, a high-HHI dummy, or the CR4. We include interaction terms between import tariffs and market concentration, i.e., $\log(tariff+1)_{pt-1} * MC_{pt-1}$ to

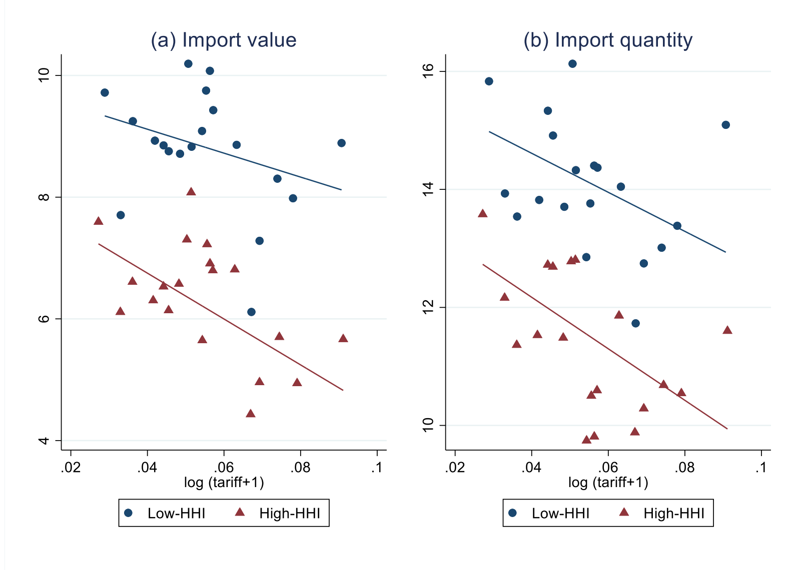

Figure 2: Input tariffs and imports: High-HHI versus low-HHI

Notes: This figure plots the relationship between input tariffs and imports (value and quantity) using data from 2007. Each dot (or triangle) represents a bin of product-level observations.

capture heterogeneous effects. Finally, we control for product fixed effects FE_p to eliminate the time-invariant differences across products, and year fixed effects FE_t to control for various macroeconomic shocks common across products. Standard errors are clustered at the product level.

The coefficient of interest is β_3, which we expect to be negative. Our hypothesis is that, relative to imported intermediate goods with low market concentration, those with high market concentration are more likely to be adversely affected by import tariffs. As previously discussed, we restrict the sample to manufacturing importers and assume that these downstream firms use imported input in their production. The dataset includes 3,053 products over the period 2000–2007.

3.3 Results

Before presenting estimation results, we first report graphical results. Figure 2 plots import value and quantity on the vertical axis against input tariffs on the horizontal axis. Panel (a) shows that import value decreases as input tariffs increase, for both high-HHI and low-HHI products. Compared to low-HHI products, high-HHI products tend to exhibit lower import value but larger tariff elasticity. Similarly, Panel (b) indicates a negative relationship between import quantity and input tariffs for both groups. Interestingly, compared to import value, import quantity appears more responsive to input tariffs. These figures use the data from 2007 (the final year of our panel), but similar patterns are observed in other years as well.

Table 2 reports the estimation results on input tariffs, imports, and market concentration. The dependent variables are the logarithm of import value in columns (1)–(4) and the logarithm of import quantity in columns (5)–(8). A one-percentage-point reduction in input tariffs – from 10% to 9% – is associated with a 1.5% increase in import value (column (1)) and a 5.9% increase in import quantity (column (5)).[18] Although the estimated

[18] For example, in column (1), the effect on import value, when the input tariff decreases from 10% to 9% is calculated as $[\ln(0.09 + 1) - \ln(0.10 + 1)] \times (-1.696) = 0.015$.

Table 2: Input tariffs, imports, and market concentration

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. Var:	log(import value)				log(import quantity)			
L.log(tariff+1)	-1.696	-0.306	-0.335	2.616	-6.476***	-3.968*	-4.246**	-0.151
	(1.640)	(1.868)	(1.723)	(2.158)	(1.949)	(2.206)	(2.029)	(2.512)
L.HHI		-0.292				-0.249		
		(0.184)				(0.215)		
L.log(tariff+1)*L.HHI		-2.042				-4.025*		
		(2.091)				(2.427)		
L.high_HHI			-0.0212				0.0354	
			(0.0639)				(0.0744)	
L.log(tariff+1)*L.high_HHI			-1.487*				-2.512***	
			(0.812)				(0.941)	
L.CR4				0.0779				0.177
				(0.119)				(0.138)
L.log(tariff+1)*L.CR4				-4.584**				-6.778***
				(1.814)				(2.111)
Product FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	19,534	19,534	19,534	19,534	19,534	19,534	19,534	19,534
adj. R^2	0.883	0.883	0.883	0.883	0.882	0.883	0.882	0.882

Notes: The number of HS 6-digit products is 3,053, and the sample period covers 2000-2007. The dependent variables are the logarithms of ordinary import (value and quantity) of raw materials and intermediate goods into China at the product level. Tariffs refer to input tariffs. Standard errors are clustered at the product level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

coefficients on input tariffs are negative but not statistically significant in some specifications, the interaction terms between input tariffs and market concentration are negative and statistically significant in many cases. This suggests that the effects of input tariffs on import value and quantity depend on market concentration. Specifically, column (3) shows that a one-percentage-point reduction in input tariffs increases the import value of high-HHI products by approximately 1.6%.[19] The effect is even larger for import quantity: as shown in column (6), the same tariff reduction leads to a 6.2% increase in the import quantity of high-HHI products. Moreover, the heterogeneous effects are even more pronounced when using CR4 as the measure of concentration. These results suggest that imported intermediate goods in more concentrated markets are more sensitive to tariff changes.

Our analysis so far focuses on raw materials and intermediate goods and excludes capital goods. However, as shown in Appendix Table A2, the estimation results remain robust and quantitatively similar even when capital goods are also treated as intermediate goods.

[19] It is calculated as $[\ln(0.09 + 1) - \ln(0.10 + 1)] \times (-1.822) = 0.016$.

4 Conclusion

This paper has analyzed a vertical oligopoly model to examine the effect of tariffs on trade elasticity and welfare when vertically related sectors are oligopolistic. Our central contribution is to provide a better understanding of the mechanism through which tariffs affect the above key equilibrium variables from the long-run perspectives. To show this mechanism, we resort to constant-elasticity demand throughout our analysis to gain tractability, but allow for free entry to disentangle the effect of tariffs on the intensive and extensive margins. We find that increased tariffs decrease these margins simultaneously in vertically related sectors, raising the trade elasticity relative to that in competitive sectors. This difference also gives rise to a different effect of tariffs on welfare, causing severe welfare losses under vertical oligopoly relative to those under perfect competition. Furthermore, we also conduct the empirical analysis for the relationship between market concentration and trade elasticities predicted by the model, and find evidence that imported intermediate goods in more concentrated markets are more sensitive to tariff changes.

While many papers have demonstrated the importance of vertical specialization and GVCs in accounting for recent trade flows, most existing work assumes perfectly competitive firms in a multistage production process. In reality, however, globalization leads to the increasing strategic interdependence among large firms that are unlikely to be of measure zero relative to the markets: only a few large firms participate in international trade and the markets are becoming more concentrated toward these firms over time, suggesting that market structure is more characterized by oligopoly. The present paper tries to fill this significant gap in the literature by exploring market-based interactions between vertically related sectors. To avoid the complexity of vertical oligopoly where strategic interdependences among producers inherently exist, we have employed restrictive conditions not only for the demand side but also for the supply side (e.g., two-stage production with representative producers and one-to-one technology). The extent to which our results can be generalized is crucial for understanding the role of strategically interdependent producers in vertical specialization and GVCs and its related trade policy issues, which we leave to future research.

A Appendix

This appendix presents detailed expressions of theoretical results and provides additional evidence that supports the model setting. To achieve the first goal, it is useful to derive the comparative statics on the input demand. Differentiating (5) with respect to its arguments,

$$
\begin{aligned}
g_X(X,M,\tau,t) &= \frac{P'(.)(M+1+\rho)}{M\tau t} < 0, \qquad & g_M(X,M,\tau,t) &= -\frac{P(.)(1+\rho)}{M^2\tau} > 0,\\
g_\tau(X,M,\tau,t) &= -\frac{P(.)(M+1+\rho)}{M\tau^2} < 0, \qquad & g_t(X,M,\tau,t) &= -\frac{QP'(.)(M+1+\rho)}{M\tau t} > 0,\\
g_{XX}(X,M,\tau,t) &= \frac{P''(.)(M+1+\rho)}{M\tau t^2} > 0, \qquad & g_{XM}(X,M,\tau,t) &= -\frac{P'(.)(1+\rho)}{M^2\tau t} < 0,\\
g_{X\tau}(X,M,\tau,t) &= -\frac{P'(.)(M+1+\rho)}{M\tau^2} > 0, \qquad & g_{Xt}(X,M,\tau,t) &= -\frac{P'(.)(1+\rho)(M+1+\rho)}{M\tau t^2} > 0,
\end{aligned}
\tag{A.1}
$$

where the signs of (A.1) follow from noting $-2<\rho<-1$ ($P''(.)>0$) under the constant-elasticity demand.

A.1 Proof of Lemma 2

(i) Differentiating (8) with respect to τ and using $Q = X/t$, (A.1) as well as (7) in Lemma 1,

$$
\begin{aligned}
\frac{\partial Q}{\partial \tau} &= \frac{1}{1+\rho}\frac{Q}{\tau} < 0, \qquad & \frac{\partial q}{\partial \tau} &= \frac{1}{M}\frac{\partial Q}{\partial \tau} = \frac{1}{1+\rho}\frac{q}{\tau} < 0,\\
\frac{\partial X}{\partial \tau} &= \frac{1}{1+\rho}\frac{X}{\tau} < 0, \qquad & \frac{\partial x}{\partial \tau} &= \frac{1}{N}\frac{\partial X}{\partial \tau} = \frac{1}{1+\rho}\frac{x}{\tau} < 0.
\end{aligned}
$$

Similarly, differentiating (8) with respect to t and using (A.1),

$$
\begin{aligned}
\frac{\partial Q}{\partial t} &= \frac{1}{1+\rho}\frac{Q}{t} < 0, \qquad & \frac{\partial q}{\partial t} &= \frac{1}{M}\frac{dQ}{dt} = \frac{1}{1+\rho}\frac{q}{t} < 0,\\
\frac{\partial X}{\partial t} &= \frac{2+\rho}{1+\rho}\frac{X}{t} < 0, \qquad & \frac{\partial x}{\partial t} &= \frac{1}{N}\frac{\partial X}{\partial t} = \frac{2+\rho}{1+\rho}\frac{x}{t} > 0.
\end{aligned}
\tag{A.2}
$$

(ii) Differentiating (8) with respect to N and using (A.1),

$$
\begin{aligned}
\frac{\partial Q}{\partial N} &= \frac{x/t}{N+1+\rho} > 0, \qquad & \frac{\partial q}{\partial N} &= \frac{1}{M}\frac{\partial Q}{\partial N} = \frac{x/t}{M(N+1+\rho)} > 0,\\
\frac{\partial X}{\partial N} &= \frac{x}{N+1+\rho} > 0, \qquad & \frac{\partial x}{\partial N} &= \frac{1}{N}\frac{\partial X}{\partial N} - \frac{x}{N} = -\frac{(N+\rho)x}{N(N+1+\rho)} < 0.
\end{aligned}
\tag{A.3}
$$

Similarly, differentiating (8) with respect to M and using (6),[20]

$$
\begin{aligned}
\frac{\partial Q}{\partial M} &= \frac{q}{M+1+\rho} > 0, \qquad & \frac{\partial q}{\partial M} &= \frac{1}{M}\frac{dQ}{dM} - \frac{q}{M} = -\frac{(M+\rho)q}{M(M+1+\rho)} < 0,\\
\frac{\partial X}{\partial M} &= \frac{tq}{M+1+\rho} > 0, \qquad & \frac{\partial x}{\partial M} &= \frac{1}{N}\frac{\partial X}{\partial M} = \frac{tq}{N(M+1+\rho)} > 0.
\end{aligned}
\tag{A.4}
$$

[20]Note that (A.3) and (A.4) has direct implications of Lemma 3. For the quantity in the downstream sector, for example, we get

$$\lim_{M\to\infty, N\to\infty} \frac{dQ}{dM} = \frac{dQ}{dN} = \frac{dq}{dM} = \frac{dq}{dN} = 0.$$

A.2 Proof of Proposition 1

(i) We first show the derivations of impacts of tariffs. Since $\frac{dQ}{d\tau}$ was shown in (12), we consider other variables. Taking the log and differentiating $X = tQ$ with respect to τ, we get $\frac{dX}{d\tau} = \frac{dQ}{d\tau}$. Further, using (12) and solving for the system of three equations in the main text, the impact on the number of firms is

$$\begin{aligned} \frac{dM}{d\tau} &= \mu(.)\left(\frac{2+\rho}{2(1+\rho)}\right)\frac{M}{\tau} < 0, \\ \frac{dN}{d\tau} &= \left(-\frac{1}{2} + \mu(.)\left(\frac{(2+\rho)}{(1+\rho)}\frac{(2M+1+\rho)}{4(M+1+\rho)}\right)\right)\frac{N}{\tau} < 0, \end{aligned} \tag{A.5}$$

where the signs of (A.5) follow immediately from $1+\rho < 0$ and $\mu(.) > 0$ in view of (2) and (12), respectively. Finally, taking the log and differentiating $q = \frac{Q}{M},\ x = \frac{X}{N}$, the impact on the firm-level quantity is

$$\begin{aligned} \frac{dq}{d\tau} &= -\mu(.)\left(\frac{\rho}{2(1+\rho)}\right)\frac{q}{\tau} < 0, \\ \frac{dx}{d\tau} &= \left(\frac{1}{2} + \mu(.)\left(\frac{(2+\rho) - \rho(2M+\rho)}{4(1+\rho)(M+1+\rho)}\right)\right)\frac{x}{\tau} < 0. \end{aligned} \tag{A.6}$$

The sign of the first equality in (A.6) follows from noting that $\frac{dq}{d\tau} < 0$ if and only if demand is convex ($\rho < 0$), which holds for constant-elasticity demand. The sign of the second equality in (A.6) follows from noting that $\frac{dx}{d\tau} < 0$ if and only if the value in the brackets is negative, which, using $\mu(.)$ in (13), is rewritten as

$$\frac{2M+\rho}{(2+\rho) - \rho(2M+\rho)} < -\frac{1}{1+\rho}.$$

Upon rearrangement, it can be confirmed that the inequality always holds for $-2 < \rho < -1$.

Next, we show the derivations of impacts of transport costs. Taking the log and totally differentiating (8), (10) and (11) with respect to t where $Q = X/t$, we get

$$\begin{aligned} (1+\rho)\hat{X} &= \left(\frac{1+\rho}{M+1+\rho}\right)\hat{M} + \left(\frac{1+\rho}{N+1+\rho}\right)\hat{N} + (2+\rho)\hat{t}, \\ (2+\rho)\hat{X} &= 2\hat{M} + (2+\rho)\hat{t}, \\ (2+\rho)\hat{X} &= \left(\frac{1+\rho}{M+1+\rho}\right)\hat{M} + 2\hat{N} + (1+\rho)\hat{t}. \end{aligned}$$

Since this is the system of three equations with three unknowns $(\hat{X}, \hat{M}, \hat{N})$, the solution to $\hat{X}$ is given by

$$\frac{dX}{dt} = \eta(M, N, \rho)\left(\frac{2+\rho}{1+\rho}\right)\frac{X}{t} < 0, \tag{A.7}$$

where

$$\eta(M, N, \rho) \equiv \frac{2(2M+1+\rho)(N+1+\rho) - (1+\rho)^2\left(\frac{2M+\rho}{2+\rho}\right)}{(2M+\rho)(2N+\rho) - (2+\rho)}. \tag{A.8}$$

Using (A.7), it is straightforward to have the impacts on the number of firms and firm-level quantity similar to (A.5) and (A.6). Note that the trade elasticity with respect to transport costs in Proposition 2 directly comes from (A.7) and (A.8).

(ii) We show the derivations of (16). Noting $r = g(X, M, \tau, t)$ and totally differentiating (5) with respect to τ,

$$\frac{dr}{d\tau} = g_X(.)\frac{dX}{d\tau} + g_M(.)\frac{dM}{d\tau} + g_\tau(.),$$

where the effect of tariffs on aggregate output is decomposed as

$$\frac{dX}{d\tau} = \frac{\partial X}{\partial \tau} + \frac{\partial X}{\partial M}\frac{dM}{d\tau} + \frac{\partial X}{\partial N}\frac{dN}{d\tau}.$$

Using this decomposition for the equality above and rearranging,

$$\frac{dr}{d\tau} = \underbrace{g_X(.)\frac{\partial X}{\partial \tau} + g_\tau(.)}_{\frac{\partial r}{\partial \tau}} + \underbrace{\left(g_X(.)\frac{\partial X}{\partial M} + g_M(.)\right)}_{\frac{\partial r}{\partial M}}\frac{dM}{d\tau} + g_X(.)\frac{\partial X}{\partial N}\frac{dN}{d\tau}. \tag{A.9}$$

Note that the first two terms in (A.9) are $\frac{\partial r}{\partial \tau}$ in Lemma 2(iii), which is zero under constant-elasticity demand. Regarding the value in the brackets in the third term, using (6) and (A.4), we get

$$\begin{aligned}\frac{\partial r}{\partial M} &= g_X(.)\frac{\partial X}{\partial M} + g_M(.)\\ &= \frac{P'(.)(M+1+\rho)}{M^2\tau t}\cdot\frac{tq}{M+1+\rho} - \frac{P(.)(1+\rho)}{M^2\tau}\\ &\frac{P(.)}{M^2\tau}\left(\frac{QP'(Q)}{P(.)} - (1+\rho)\right)\\ &= 0.\end{aligned}$$

Thus, the third term is also zero under constant-elasticity demand.

A.3 Proof of Proposition 3

We first show that the *relative* pass-through is decreasing in tariffs. Taking the log and totally differentiating the first-order conditions (4) and (8) with respect to τ,

$$\begin{aligned}\frac{d\ln P}{d\ln\tau} - \frac{d\ln(r\tau)}{d\ln\tau} &= \left(\frac{1+\rho}{M+1+\rho}\right)\frac{d\ln M}{d\ln\tau} > 0,\\ \frac{d\ln r}{d\ln\tau} - \frac{d\ln(ct)}{d\ln\tau} &= \left(\frac{1+\rho}{N+1+\rho}\right)\frac{d\ln N}{d\ln\tau} > 0,\end{aligned}$$

where the sign follows from $1+\rho < 0$ and $\frac{dM}{d\tau} < 0$, $\frac{dN}{d\tau} < 0$, as seen in (A.5).

Next, we show that the *absolute* pass-through is decreasing in tariffs. Dividing both sides of the free entry conditions (10) and (11) by $\frac{Q}{M} = q$ and $\frac{X}{N} = x$, respectively, and totally differentiating them with respect to τ,

$$\begin{aligned}\frac{d(P - r\tau)}{d\tau} &= -\left(\frac{K_H}{q^2}\right)\frac{dq}{d\tau} > 0,\\ \frac{d(r - ct)}{d\tau} &= -\left(\frac{K_F}{x^2}\right)\frac{dx}{d\tau} > 0,\end{aligned}$$

where the signs follows from $\frac{dq}{d\tau} < 0$, $\frac{dx}{d\tau} < 0$, as seen in (A.6).

A.4 Effects of Transport Costs

We first show that transport costs have a positive effect on the terms-of-trade when the number of firms is fixed. Noting $r = g(X, M, \tau, t)$ that satisfy (A.1) and using (A.2), we get

$$\frac{\partial r}{\partial t} = g_X(.)\frac{\partial X}{\partial t} + g_t(.) = -\frac{XP'(.)(M+1+\rho)}{M\tau t^2}\left(\frac{\rho}{1+\rho}\right) > 0.$$

Thus, transport costs always increases the input price. Recalling that the terms-of-trade for Home are defined as $1/r$ in our model, this means that transport costs negatively affect the terms-of-trade. The difference between import tariffs and transport costs comes from whether these trade costs affect either the demand or supply side of intermediate input, as we argued when deriving the partials in (6).

Next, we show that transport costs have a negative effect on the terms-of-trade when the number of firms is endogenous via free entry. Similarly to (A.9), this effect is decomposed as

$$\frac{dr}{d\tau} = \underbrace{g_X(.)\frac{\partial X}{\partial t} + g_t(.)}_{\frac{\partial r}{\partial t}} + \underbrace{\left(g_X(.)\frac{\partial X}{\partial M} + g_M(.)\right)}_{\frac{\partial r}{\partial M}}\frac{dM}{dt} + g_X(.)\frac{\partial X}{\partial N}\frac{dN}{dt} > 0. \tag{A.10}$$

The sign of (A.10) is explained as follows. The first two terms in (16) are the same as $\frac{\partial r}{\partial t} > 0$, as seen above. On the other hand, the value in the brackets in the third term is zero, just as in (A.9). Finally, the last term is positive in light of $g_X(.) < 0$, $\frac{\partial X}{\partial N} > 0$ and $\frac{dN}{dt} < 0$, as shown in (A.1), (A.3) and Appendix A.2, respectively. From (16) and (A.10), it follows immediately that the effect of transport costs on the terms-of-trade is similar to that of import tariffs under free entry.

Finally, we show that transport costs have welfare effects similar to import tariffs. It is evident to see that the difference in welfare between a fixed number of firms and a free-entry number of firms continue to hold for transport costs. In particular, (20) is given by

$$\frac{dW}{d\tau} - \frac{\partial W}{\partial t} = (P - r)\left(\frac{\partial Q}{\partial M}\frac{dM}{dt} + \frac{\partial Q}{\partial N}\frac{dN}{dt}\right) - Q\frac{dr}{dt} < 0,$$

where the sign follows from noting (A.3) and (A.10).

A.5 Additional Evidence

Table A.1 lists the top 10 most concentrated manufacturing industries in terms of the Herfindahl-Hirschman Index (HHI), along with their corresponding upstreamness. The HHI is calculated using micro-data from China's Annual Survey of Industrial Firms (ASIF) for the year 2007, while the upstreamness measure is taken from Chor et al. (2021). The average HHI across 80 Input-Output (IO) industries is 0.009. Among the most concentrated sectors are Communications Equipment, Tobacco, and Beverages. Notably, industries with an HHI above the median value (0.01) played a significant role in the Chinese economy, accounting for 31.6% of total sales, 38.0% of exports, and 17.9% of employment in the manufacturing sector. The upstreamness index ranges from 1.963 (Lifting and Transport Equipment) to 4.788 (Ironmaking). Given that the median upstreamness value across manufacturing industries is 3.06, there is no systematic correlation between upstreamness and market concentration. As a result, highly concentrated industries can be found at both the upstream and downstream stages of the production process.

Table A.1: Top 10 concentrated industries in China, 2007

	China IO2007 industry	HHI	Upstreamness
82	Communications equipment manufacturing	0.057	2.061
24	Tobacco manufacturing	0.041	2.498
16	Beverage processing	0.032	2.629
84	Computer manufacturing	0.031	2.957
66	Lifting and transport equipment manufacturing	0.027	1.963
87	Other electronic equipment manufacturing	0.027	2.573
45	Daily-use chemical product manufacturing	0.026	2.859
58	Steelmaking	0.024	4.618
73	Rail transportation equipment manufacturing	0.023	2.520
57	Ironmaking	0.022	4.788
	All manufacturing industries: mean	0.009	3.251
	median	0.010	3.060

Source: Authors' own calculations based on the micro-data from the Annual Survey of Industrial Firms (ASIF), National Bureau of Statistics of China, and the Input-Output (IO) industry-level upstreamness index constructed by Chor et al. (2021).

Notes: The ASIF data cover industrial firms with annual sales above 5 million RMB and all state-owned enterprises, including both importers and non-importers, across 424 China's Industrial Classification (CIC) 4-digit manufacturing industries. Using a concordance table, we map CIC 4-digit industries to IO 3-digit industries. Higher values indicate greater market concentration and upstreamness.

In sum, Table A.1 and Figure 1 support the idea that oligopoly is prevalent across upstream and downstream segments of the manufacturing production chain. They also show that market concentration varies within each production group, thereby justifying the modeling of both upstream and downstream industries as oligopolistic in theoretical and empirical frameworks.

Table A.2 reports the correlation matrix of the key variables. Import value and quantity are negatively associated with both output and input tariffs, as well as with market concentration. The correlation coefficient between output tariffs and input tariffs is 0.52.

Table A.2: Correlation matrix

Variable	log(Import value)	log(Import quantity)	Number of importers	Applied (output) tariffs	Input tariff (%)	HHI	High-HHI dummy	CR4
log (Import value)	1							
log(Import quantity)	0.84	1						
Number of importers	0.35	0.24	1					
Applied (output) tariff (%)	-0.21	-0.17	-0.07	1				
Input tariff (%)	-0.24	-0.22	-0.07	0.52	1			
HHI	-0.55	-0.46	-0.30	0.10	0.15	1		
High-HHI dummy	-0.48	-0.39	-0.29	0.10	0.12	0.76	1	
CR4	-0.40	-0.32	-0.33	0.06	0.09	0.67	0.68	1

Source: Authors' compilation based on Chinese Customs data, WITS, and Brandt et al. (2017).

Table A.3 reports the estimation results when capital goods is also treated as intermediate goods. Note that our analysis focuses on raw materials and intermediate goods and excludes capital goods. Relative to Table 2, the results remain robust and quantitatively similar.

Table A.3: Input tariffs, imports, and market concentration: Including capital goods as intermediate goods

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. Var:		log(imports value)				log(imports quantity)		
L.log(tariff+1)	-1.302	-0.590	-0.206	1.704	-5.577***	-4.040**	-3.778**	-0.686
	(1.507)	(1.694)	(1.562)	(1.982)	(1.773)	(1.997)	(1.832)	(2.324)
L.HHI		-0.443***				-0.389**		
		(0.170)				(0.196)		
L.log(tariff+1)*L.HHI		-0.744				-2.253		
		(1.935)				(2.228)		
L.high_HHI			-0.0413				-0.000560	
			(0.0602)				(0.0694)	
L.log(tariff+1)*L.high_HHI			-1.309*				-2.186**	
			(0.766)				(0.887)	
L.CR4				-0.0127				0.0939
				(0.108)				(0.126)
L.log(tariff+1)*L.CR4				-3.196*				-5.273***
				(1.671)				(1.947)
Product FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	23,993	23,993	23,993	23,993	23,993	23,993	23,993	23,993
adj. R^2	0.885	0.886	0.885	0.885	0.918	0.919	0.919	0.919

Notes: The number of HS 6-digit products is 3,734, and the sample period covers 2000-2007. The dependent variables are the logarithms of ordinary import (value and quantity) of raw materials, intermediate goods, and capital goods into China at the product level. Tariffs refer to input tariffs. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

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